Chapter 23
Planning in the Game of Bridge

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Computer Programs for Games of Strategy

Connect Four: solved
Go-Moku: solved
Qubic: solved
Nine Men’s Morris: solved
Checkers: solved
Othello: better than humans
Backgammon: better than all but about 10 humans
Chess: competitive with the best humans
Bridge: about as good as mid-level humans
Computer Programs for Games of Strategy

- Fundamental technique: the minimax algorithm

\[
\text{minimax}(u) = \begin{cases} 
\max \{ \text{minimax}(v) : v \text{ is a child of } u \} & \text{if it's Max's move at } u \\
\min \{ \text{minimax}(v) : v \text{ is a child of } u \} & \text{if it's Min's move at } u 
\end{cases}
\]

- Largely “brute force”
- Can prune off portions of the tree
  - cutoff depth & static evaluation function
  - alpha-beta pruning
  - transposition tables
  - …
- But even then, it still examines thousands of game positions

For bridge, this has some problems …
How Bridge Works

- Four players; 52 playing cards dealt equally among them
- Bidding to determine the trump suit
  - Declarer: whoever makes highest bid
  - Dummy: declarer’s partner
- The basic unit of play is the trick
  - One player leads; the others must follow suit if possible
  - Trick won by highest card of the suit led, unless someone plays a trump
  - Keep playing tricks until all cards have been played
- Scoring based on how many tricks were bid and how many were taken
Game Tree Search in Bridge

- Bridge is an *imperfect information* game
  - Don't know what cards the others have (except the dummy)
  - Many possible card distributions, so many possible moves
- If we encode the additional moves as additional branches in the game tree, this increases the branching factor $b$
- Number of nodes is exponential in $b$
  - worst case: about $6 \times 10^{44}$ leaf nodes
  - average case: about $10^{24}$ leaf nodes

- A chess game may take several hours
- A bridge game takes about 1.5 minutes

Not enough time to search the game tree
Reducing the Size of the Game Tree

- One approach: HTN planning
  - Bridge is a game of planning
  - The declarer plans how to play the hand
  - The plan combines various strategies (ruffing, finessing, etc.)
  - If a move doesn’t fit into a sensible strategy, it probably doesn’t need to be considered

- Write a planning procedure similar to TFD (see Chapter 11)
  - Modified to generate game trees instead of just paths
  - Describe standard bridge strategies as collections of methods
  - Use HTN decomposition to generate a game tree in which each move corresponds to a different strategy, not a different card

<table>
<thead>
<tr>
<th></th>
<th>Brute-force search</th>
<th>HTN-generated trees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worst case</td>
<td>$\approx 6 \times 10^{44}$ leaf nodes</td>
<td>$\approx 305,000$ leaf nodes</td>
</tr>
<tr>
<td>Average case</td>
<td>$\approx 10^{24}$ leaf nodes</td>
<td>$\approx 26,000$ leaf nodes</td>
</tr>
</tbody>
</table>
Methods for Finessing

- **Finesse(P₁; S)**
- **LeadLow(P₁; S)**
- **PlayCard(P₁; S, R₁)**
- **EasyFinesse(P₂; S)**
- **StandardFinesse(P₂; S)**
- **BustedFinesse(P₂; S)**
- **FinesseTwo(P₂; S)**
- **FinesseThree(P₃; S)**
- **FinesseFour(P₄; S)**
- **StandardFinesseTwo(P₂; S)**
- **StandardFinesseThree(P₃; S)**
- **PlayCard(P₂; S, R₂)**
- **PlayCard(P₃; S, R₃)**
- **PlayCard(P₄; S, R₄)**
- **PlayCard(P₄; S, R₄')**

**Task**: Finessing

**Method**: Game play

**Time Ordering**: Possible moves by 1st opponent

**Dummy**: Game play

**1st Opponent**: Declarer

**2nd Opponent**: Declarer
Instantiating the Methods

Us: East declarer, West dummy
Opponents: defenders, South & North
Contract: East – 3NT
On lead: West at trick 3

East: ♠KJ74
West: ♠A2
Out: ♠QT98653

Us: East declarer, West dummy
Opponents: defenders, South & North
Contract: East – 3NT
On lead: West at trick 3

Us: East declarer, West dummy
Opponents: defenders, South & North
Contract: East – 3NT
On lead: West at trick 3
Generating Part of a Game Tree

The red boxes are the leaf nodes.
Game Tree Generated using the Methods

... later stratagems ...

FINESSE

W—★2
+270.73

N—★2
0.9854
+265

E—★J
+265

S—★Q
0.5
S—★5
0.5

---

N—★Q
0.0078
+630

E—★K
+630

S—★3
---

N—☐3
0.0078
+600

E—★K
+600

S—★3
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CASH OUT

W—★A
+600

N—★3
+600

E—★4
+600

S—★5
+600

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Implementation

- Stephen J. Smith, then a PhD student at U. of Maryland
  - Wrote a procedure to plan declarer play
- Incorporated it into *Bridge Baron*, an existing commercial product
  - This significantly improved *Bridge Baron's* declarer play
  - Won the 1997 world championship of computer bridge
- Since then:
  - Stephen Smith is now Great Game Products’ lead programmer
  - He has made many improvements to *Bridge Baron*
    » Proprietary, I don’t know what they are
  - *Bridge Baron* was a finalist in the 2003 and 2004 computer bridge championships
    » I haven’t kept track since then
Other Approaches

- Monte Carlo simulation:
  - Generate many random hypotheses for how the cards might be distributed
  - Generate and search the game trees
    » Average the results
  - This can divide the size of the game tree by as much as $5.2 \times 10^6$
    » $(6 \times 10^{44})/(5.2 \times 10^6) = 1.1 \times 10^{38}$
      - still quite large
    » Thus this method by itself is not enough
Other Approaches (continued)

- AJS hashing - Applegate, Jacobson, and Sleator, 1991
  - Modified version of transposition tables
    » Each hash-table entry represents a set of positions that are considered to be equivalent
    » Example: suppose we have ♠AQ532
      • View the three small cards as equivalent: ♠Aqxxx
  - Before searching, first look for a hash-table entry
    » Reduces the branching factor of the game tree
    » Value calculated for one branch will be stored in the table and used as the value for similar branches
- GIB (1998-99 computer bridge champion) used a combination of Monte Carlo simulation and AJS hashing
- Several current bridge programs do something similar
Top contenders in computer bridge championships, 1997–2004

<table>
<thead>
<tr>
<th>Year</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>Bridge Baron</td>
<td>Q-Plus</td>
<td>Micro Bridge</td>
<td>Meadowlark</td>
</tr>
<tr>
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<td>GIB</td>
<td>Q-Plus</td>
<td>Micro Bridge</td>
<td>Bridge Baron</td>
</tr>
<tr>
<td>1999</td>
<td>GIB</td>
<td>WBridge5</td>
<td>Micro Bridge</td>
<td>Bridge Buff</td>
</tr>
<tr>
<td>2000</td>
<td>Meadowlark</td>
<td>Q-Plus</td>
<td>Jack</td>
<td>WBridge5</td>
</tr>
<tr>
<td>2001</td>
<td>Jack</td>
<td>Micro Bridge</td>
<td>WBridge5</td>
<td>Q-Plus</td>
</tr>
<tr>
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<td>Jack</td>
<td>Wbridge5</td>
<td>Micro Bridge</td>
<td>?</td>
</tr>
<tr>
<td>2003</td>
<td>Jack</td>
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<td>WBridge5</td>
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</tr>
</tbody>
</table>

I haven’t kept track since 2004

For more information see [http://www.jackbridge.com/ewkprt.htm](http://www.jackbridge.com/ewkprt.htm)