# Percentages for Bidding Games, Small Slams, and Grand Slams at Duplicate Teams 

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This article describes a method for deciding when to bid games, small slams, and grand slams in duplicate team matches. For bidding games and small slams the method is well known and depends on the probability P that the contract will make. For bidding grand slams our method depends not only on P, but also on the assessment of how likely the opponents stop in either game or small slam.

Deciding to bid a game is mathematically equivalent to deciding to participate in a lottery. Consider the simple lottery where a ball is selected at random from an urn containing red and white balls. If the selected ball is red you win $\$ 2$ and if it is white you lose $\$ 1$. Let P represent the probability that a red ball is selected. For what values of P will it be profitable for you to play the lottery? Since your expected winnings are $\$ 2$ * $\mathrm{P}-\$ 1$ * $(1-\mathrm{P})$, the equation describing your breakeven point is:

$$
\$ 2 * P-\$ 1 *(1-P)=0
$$

which, when simplified, becomes $3 \mathrm{P}=1$ or $\mathrm{P}=1 / 3$. More generally, to find the breakeven point you need to set the equation representing your expected winnings equal to 0 and then solve for P .

What this means is that if the collection of the balls in the urn is such that the probability of picking a red ball is greater than or equal to $1 / 3$, you will not lose money in the long run. Using an informal notation of percentages instead of the formal notation of probability, we can say that you will be profitable when your chances of picking a red ball are 33\% or better.

We apply the above method to duplicate team scoring as follows. Let's say you bid 4 of a major, non-vulnerable, and make it for +420 and your opponents bid 3 of a major making 4 for +170 . Your IMPGain is 6 . If you bid 4 of a major non-vulnerable going down one for -50 and your opponents score +140 , your IMPLoss is 5 . Let P represent the probability of bidding a major suit game when non-vulnerable. Then the breakeven point for bidding the game is the solution to the equation:

$$
[\text { IMPGain * P ] - [ IMPLoss * }(1-\mathrm{P})]=0
$$

Simplifying, we get, $6 \mathrm{P}-5(1-\mathrm{P})=0$, or $11 \mathrm{P}=5$, or $\mathrm{P}=5 / 11$ (= 0.45 or $45 \%$ ).
So by bidding a $45 \%$ or better non-vulnerable major suit game you will break even in the long run. For the sake of completeness, we give in Table 1, the percentages (rounded) for bidding game and slam contracts, both vulnerable and non-vulnerable (all these values are already available in the literature). The IMPs are calculated for contracts making
without overtricks, or down one undoubled. Notice that minor suit small slams can be bid a tad more aggressively (48\%) than major suit small slams (50\%). For NT games and slams, the percentages are the same as those for a major suit.

| Contract | Non-Vulnerable | Vulnerable |
| :---: | :---: | :---: |
| Major suit game | $\begin{gathered} \text { IMP gain }=6 \\ \mathbf{4 5 \%}(\mathrm{P}=5 / 11) \\ \text { IMP loss }=5 \end{gathered}$ | $\begin{gathered} \text { IMP gain }=10 \\ \mathbf{3 8 \%}(\mathrm{P}=6 / 16) \\ \text { IMP loss }=6 \end{gathered}$ |
| Minor suit game | $\begin{gathered} \text { IMP gain }=6 \\ \mathbf{4 5 \%}(\mathrm{P}=5 / 11) \\ \text { IMP loss }=5 \end{gathered}$ | $\begin{gathered} \text { IMP gain }=10 \\ \mathbf{3 8 \%}(\mathrm{P}=6 / 16) \\ \text { IMP loss }=6 \end{gathered}$ |
| Major suit small slam | $\begin{array}{r} \text { IMP gain }=11 \\ \mathbf{5 0 \%}(\mathrm{P}=11 / 22) \\ \text { IMP loss }=11 \end{array}$ | $\begin{gathered} \text { IMP gain }=13 \\ \mathbf{5 0 \%}(\mathrm{P}=13 / 26) \\ \text { IMP loss }=13 \end{gathered}$ |
| Minor suit small slam | $\begin{array}{r} \text { IMP gain }=11 \\ \mathbf{4 8 \%}(\mathrm{P}=10 / 21) \\ \text { IMP loss }=10 \end{array}$ | $\begin{array}{r} \text { IMP gain }=13 \\ \mathbf{4 8 \%}(\mathrm{P}=12 / 25) \\ \text { IMP loss }=12 \end{array}$ |
| Major suit grand when small slam is certain | $\begin{array}{r} \text { IMP gain }=11 \\ \mathbf{5 6 \%}(\mathrm{P}=14 / 25) \\ \text { IMP loss }=14 \end{array}$ | $\begin{array}{r} \text { IMP gain }=13 \\ \mathbf{5 7 \%}(\mathrm{P}=17 / 30) \\ \text { IMP loss }=17 \end{array}$ |
| Minor suit grand when small slam is certain | $\begin{array}{r} \text { IMP gain }=11 \\ \mathbf{5 6 \%}(\mathrm{P}=14 / 25) \\ \text { IMP loss }=14 \end{array}$ | $\begin{array}{r} \text { IMP gain }=13 \\ \mathbf{5 5 \%}(\mathrm{P}=16 / 29) \\ \text { IMP loss }=16 \end{array}$ |

## Table 1

## Grand Slam Bidding

The folklore suggests that a grand slam ought to be bid when "you can count 13 tricks." While this is sound advice at rubber bridge, at duplicate teams our method gives a more optimistic picture.

We first consider the case of a minor suit non-vulnerable grand slam where our decision is whether to stop in a small slam or bid the grand slam. Let $P$ denote the probability that the grand slam makes. Under the assumption that the opponents will surely bid the small slam, our basic equation is:

$$
11 \mathrm{P}-14(1-\mathrm{P})=0 \text { or } 25 \mathrm{P}-14=0, \text { or } \mathrm{P}=14 / 25
$$

If the opponents stop in game instead of bidding a small slam, then the number of IMPs gained by making the grand slam is $14(1440-440=1000$ points $=14$ IMPs). In such a
situation, however, you could have ensured a sure gain (11 IMPs) by stopping in a small slam. How does one take this into account? One needs to compute the Net IMPGain in stretching to bid the grand, which is only 3 IMPS (= $14-11$ ). The IMPLoss for going one down in a minor suit non-vulnerable grand when the opponents stop in game is 10 IMPs (= $420+50$ ). The basic equation is:

$$
\begin{aligned}
& \text { [Net IMPGain * P] - IMPLoss * }(1-\mathrm{P})=0 \text { or } \\
& 3 \mathrm{P}-10(1-\mathrm{P})=0 \text {, or } 13 \mathrm{P}-10=0
\end{aligned}
$$

Since we do not know a priori what the opponents will do, we have to assess the likelihood of them stopping at a small slam. Let Q denote that probability. Then ( $1-\mathrm{Q}$ ) denotes the probability that the opponents will bid game (we exclude the case where the opponents do not reach game, although this is also a distinct possibility in practice). By combining the two cases where the opponents stop in small slam or game, our breakeven equation for bidding a minor suit non-vulnerable grand slam becomes:

$$
\mathrm{Q} *[25 \mathrm{P}-14]+(1-\mathrm{Q}) *[13 \mathrm{P}-10]=0
$$

For a major suit non-vulnerable grand slam, the Net IMPGain when opponents stop in game is 3 IMPs (= $14-11$ ), the IMPLoss when opponents stop in game and the grand goes one down is 11 IMPs ( $=480+50$ ), and the breakeven equation is:

$$
\mathrm{Q} *[25 \mathrm{P}-14]+(1-\mathrm{Q}) *[3 \mathrm{P}-11(1-\mathrm{P})]=0
$$

Unlike the other equations, the breakeven value of P will depend on the value of Q . For major and minor suits when non-vulnerable, the breakeven value of $P$ is given for several different choices of Q in Table 2.

| Q = Prob. that opponents <br> bid a small slam | P values for minor suit <br> grand (non-vulnerable) | P values for major suit <br> grand (non-vulnerable) |
| :---: | :---: | :---: |
| 0.0 | $77 \%(0.77)$ | $79 \%(0.79)$ |
| 0.25 | $69 \%(0.69)$ | $70 \%(0.70)$ |
| 0.50 | $63 \%(0.63)$ | $64 \%(0.64)$ |
| 0.75 | $59 \%(0.59)$ | $60 \%(0.60)$ |
| 1.0 | $56 \%(0.56)$ | $56 \%(0.56)$ |

## Table 2

If the opponents stop in game, you will break even over the long run by bidding a $77 \%$ or better minor suit grand slam whereas if the opponents are sure to be in small slam, then you will break even by bidding a $56 \%$ or better grand slam.

For a minor suit vulnerable grand slam, the IMPs gained by making the grand when opponents stop in game is 17 ( $2140-640=1500$ points = 17 IMPs $)$, the IMPs gained by stopping in a small slam is 13 ( $1390-640=750$ points $=13$ IMPs), and the Net

IMPGain is 4 IMPs (= $17-13$ ). The IMPLoss when opponents stop in game and the grand goes one down is 12 IMPs ( $620+100=720$ points $=12$ IMPs) and the breakeven equation is:

$$
\mathrm{Q} *[29 \mathrm{P}-16]+(1-\mathrm{Q}) *[4 \mathrm{P}-12(1-\mathrm{P})]=0
$$

For a major suit vulnerable grand slam the Net IMPGain is 4 IMPs (= $17-13$ ), the IMPLoss when opponents stop in game is 13 IMPs $(=680+100)$, and the breakeven equation is:

$$
\mathrm{Q} *[30 \mathrm{P}-17]+(1-\mathrm{Q}) *[4 \mathrm{P}-13(1-\mathrm{P})]=0
$$

The values of P for several different choices of Q are given in Table 3 .

| Q = Prob. that opponents <br> bid a small slam | P values for minor suit <br> grand (vulnerable) | P values for major suit <br> grand (vulnerable) |
| :---: | :---: | :---: |
| 0.0 | $75 \%(0.75)$ | $76 \%(0.76)$ |
| 0.25 | $68 \%(0.68)$ | $69 \%(0.69)$ |
| 0.50 | $62 \%(0.62)$ | $64 \%(0.64)$ |
| 0.75 | $58 \%(0.58)$ | $60 \%(0.60)$ |
| 1.0 | $55 \%(0.55)$ | $57 \%(0.57)$ |

## Table 3

Of course, it is not always possible to find lines of play whose percentages match up exactly with those given in the tables; in practice you just try and get as close to them as possible. Some players like to work with the odds instead of percentages. It is easy to convert our analysis to odds, which are defined as follows:

Odds in favor of an event $=p /(1-p)$ where $p$ is the probability of the event.
Our analysis suggests that the decision to stretch to a grand slam should be based, in part, on your assessment of whether the opponents will bid a small slam or not - the more likely they are to bid a small slam, the more aggressive you should be in bidding a grand slam. The variable Q quantifies this decision, giving a range of P values of $56 \%$ (opponents certain to be in small slam) to 79\% (opponents certain to stop only in game). The notion of bidding a grand slam "when you can count 13 tricks" appears a bit too conservative at duplicate teams, especially when you have knowledge of your opponents' bidding methods and styles. The method given here does not take into account other considerations such as "state of the match" because that is harder to model and quantify.

When you consider bidding a grand slam, it is best to think of the percentages in scenarios such as a suit breaking 3-2 (68\%), or one of two suits breaking 3-3 (59\%), or needing to establish a long card in a 5-2 suit that requires a 3-3 or 4-2 break (84\%), or needing to establish a long card in a 5-1 suit that requires a $4-3$ break (62\%), etc. You should avoid bidding a grand slam on a scenario such as one of two finesses working
correctly. Although this succeeds $75 \%$ of the time, you can take the wrong finesse first and in a grand slam when a finesse fails you are down one already, so the status of the other finesse succeeding or failing becomes irrelevant.

In conclusion, an $84 \%$ grand slam (based on a suit splitting 3-3 or 4-2) should always be bid, a $68 \%$ grand slam (based on a suit splitting 3-2) should be bid when there is more than an even chance that the opponents will bid a small slam, and if you are certain that the opponents will bid a small slam, then you will break even over the long run by bidding a $56 \%$ grand slam. We do concede, however, that the percentages cannot always be known during the bidding.

We conclude with a couple of comments that put these mathematical percentages in perspective.

Bob Hamman's simple rule for slam bidding: "A good slam is one that makes."
Prabhakar Balakrishnan's rule for game bidding: "On hands where there 'is going to be some play,' the mathematical percentages can be pushed way down, especially for bidding games. If there is no easy path to defeating a contract, the advantage shifts in favor of declarer because of the ability to look at the weakness or strength of the combined hands. Bidding close games puts a lot of pressure on the opponents at IMP scoring."

## About the authors:

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