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BRIDGE AND PROBABILITY

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Aims

Bridge

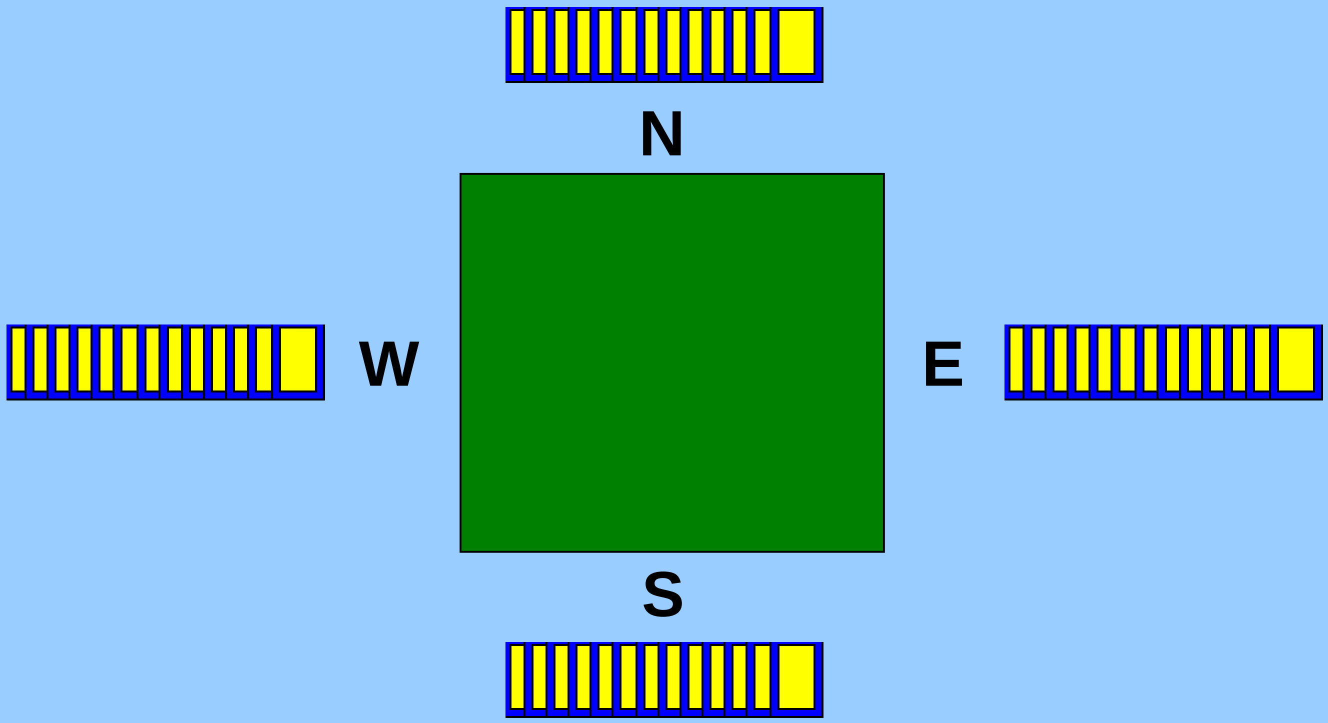
1) Introduction to Bridge

Probability

2) Number of Bridge hands
3) Odds against a Yarborough
4) Prior probabilities: Suit-Splits and Finesse
5) Combining probabilities: Suit-Splits and Finesse
6) Posterior probabilities: Suit-Splits

1) Introduction to Bridge – The Basics

- Partnership game with 4 players
- 13 cards each



Introduction to Bridge - The Auction

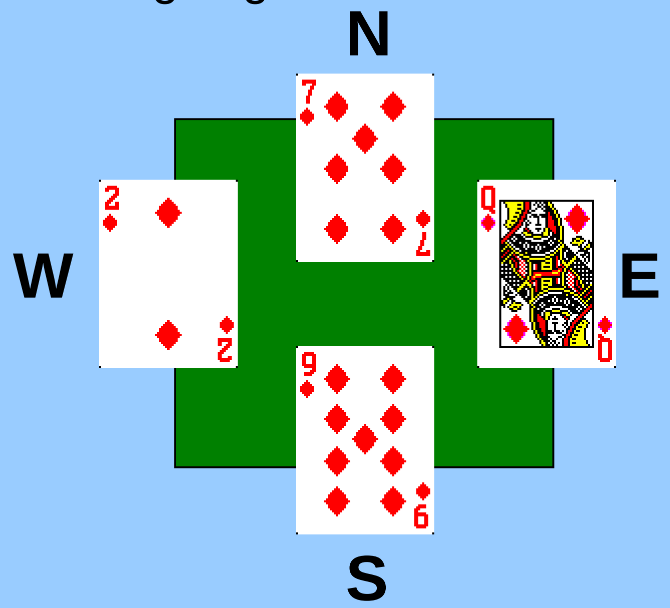
- Bid how many “tricks” you predict you and your partner can make
- Bidding boxes
- “Declarer”, “Dummy” and the “Defenders”

Bidding Box

Introduction to Bridge
- The Play

- "Dummy" hand
- One card played at a time going clockwise

- "Trick" :



- "Follow suit"
- "Trumps"
- Winner begins the next trick

2) Number of Bridge hands - The Basics

- The number of ways you can receive 13 cards from 52 is:

52	X	51	X	……………………..	X	40
possibilities for the 1st card		possibilities for the 2nd card				possibilities for the 13th card

Number of Bridge hands - Combinations

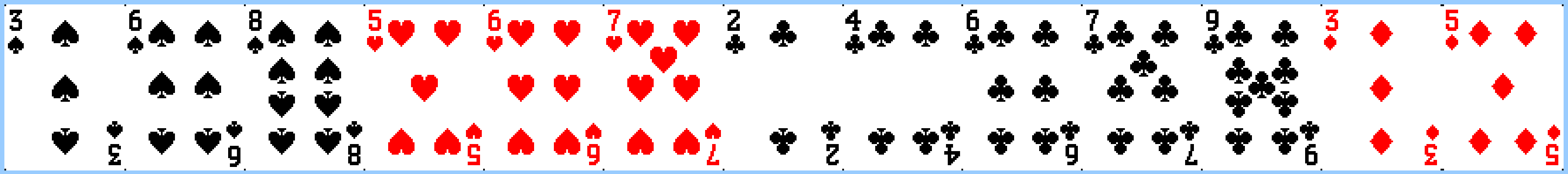
- Order does not matter
- Therefore, divide by 13! (or in general r!)

$$\frac{n \times (n-1) \times (n-2) \times \ldots \times (n - r + 1)}{r!} = {}_nC_r = \frac{n!}{(n-r)!\,r!}$$

- ${}_{52}C_{13} = \dfrac{52!}{39!13!} = $ 635,013,559,600

3) Odds against a Yarborough - The Basics

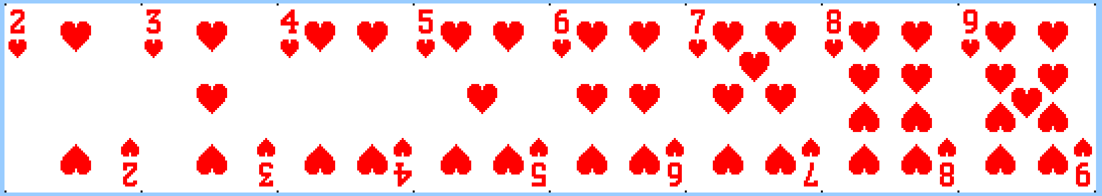
- A Yarborough = A hand containing no card higher than a 9



- Earl of Yarborough

- Number of cards no higher than 9 is:

8 in each suit:



X 4 suits = 32

Odds against a Yarborough - The Odds

- P(Yarborough) = $\dfrac{{}_{32}C_{13}}{{}_{52}C_{13}}$

- Odds against → $\dfrac{P(\bar{A})}{P(A)}$

Note $P(\bar{A})$ = complement of A
= P(not A)

- Odds against being dealt a Yarborough are 1827 to 1

4.1) Prior Probabilities – Suit-split

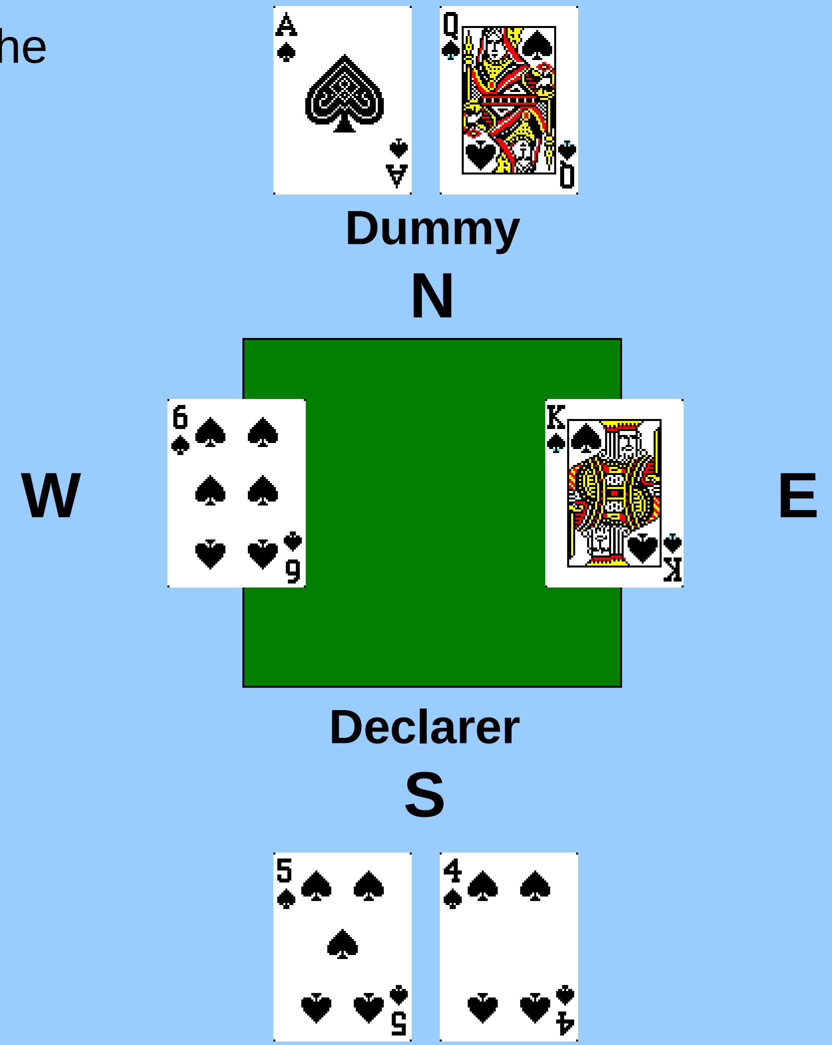
- Suit split = The division of cards between the 2 defenders in 1 suit

- P(3-3 split) = $\dfrac{{}_{6}C_{3} \;\; X \;\; {}_{20}C_{10}}{{}_{26}C_{13}}$ = 0.3553

Split	Probability
3-3	0.3553
4-2 & 2-4	0.4845
5-1 & 1-5	0.1453
6-0 & 0-6	0.0149

4.2) Prior Probabilities - Finesse

- When East has the King, the Finesse always loses



Prior Probabilities - Finesse

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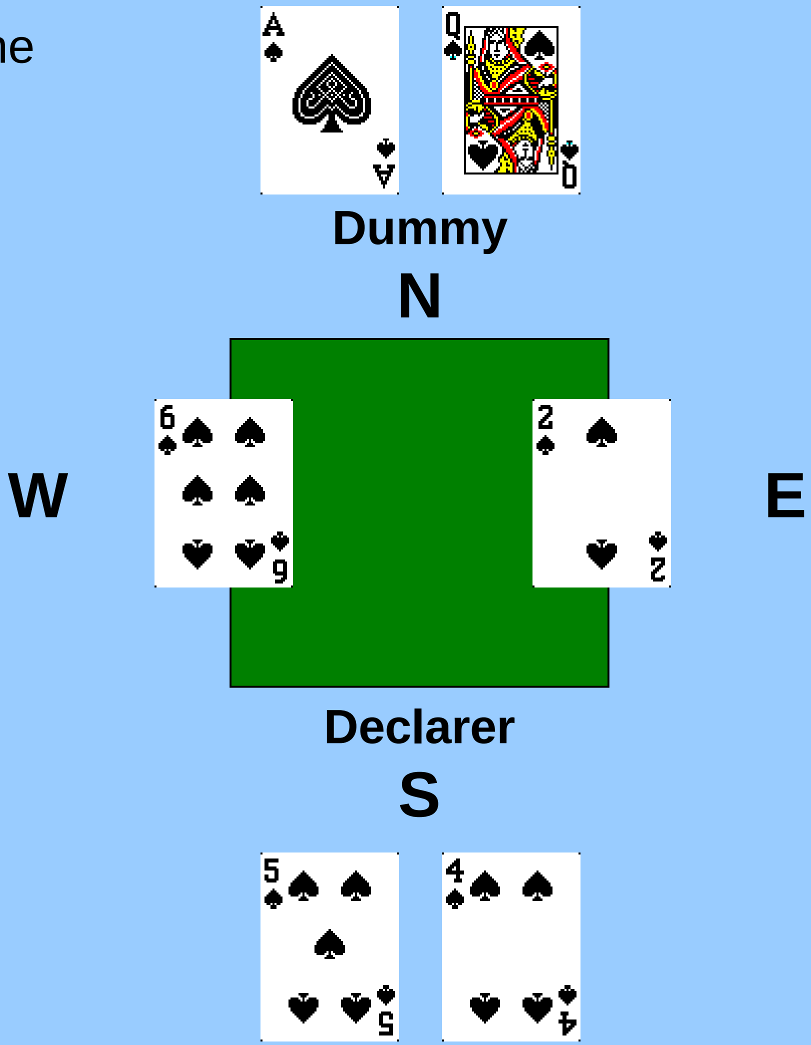

- However, when West has the King, the Finesse always wins

Prior Probabilities - Finesse

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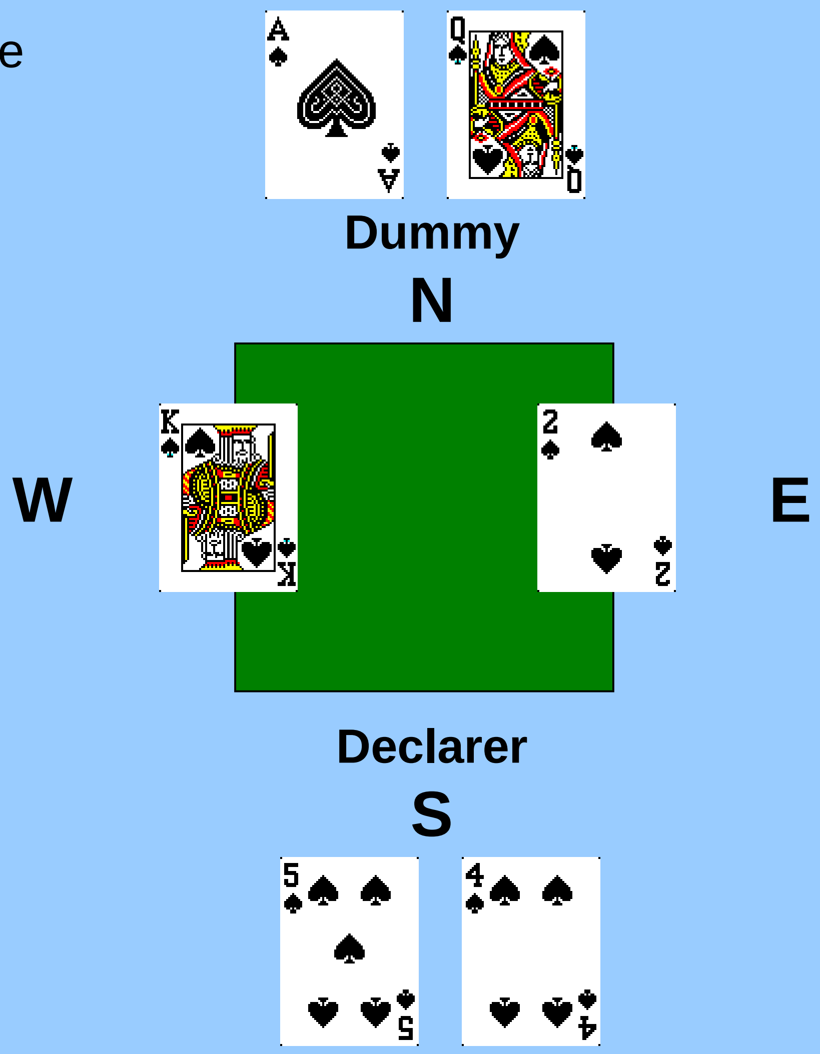
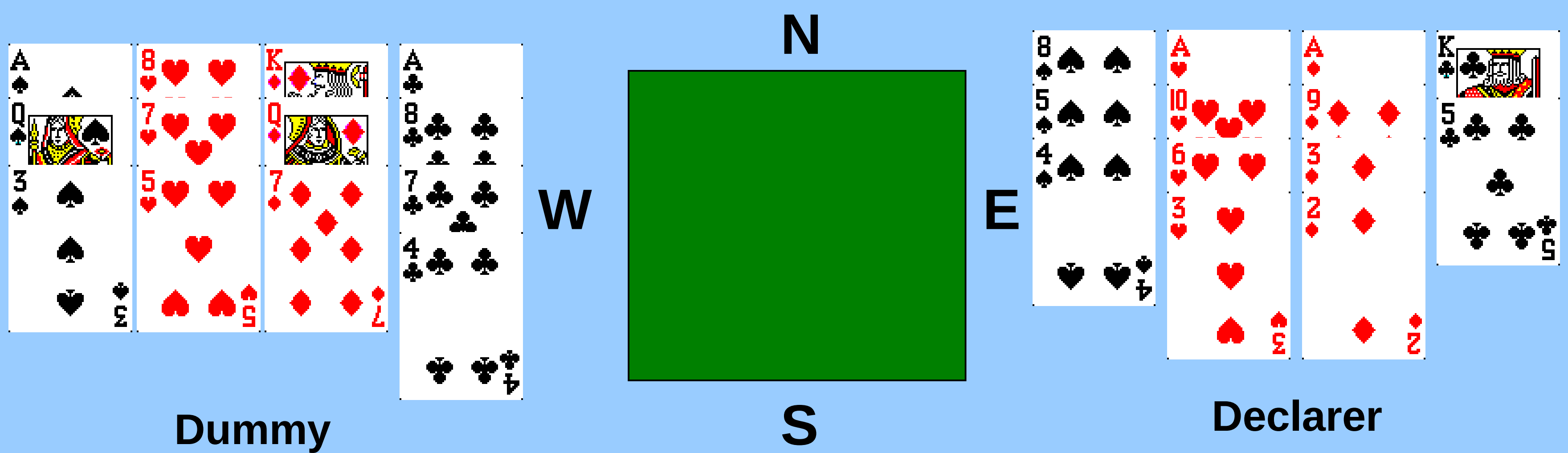


- However, when West has the King, the Finesse always wins

- Therefore the Finesse has Probability 0.5 of succeeding

5) Combining Probabilities - Intersection and Union



Suit	Definite tricks	Possible extra tricks
Spades	1 (Ace)	1 (if finesse the Queen)
Hearts	1 (Ace)	0
Diamonds	3 (Ace, King, Queen)	1 (if defenders have a 3-3 split)
Clubs	2 (Ace, King)	0
Total	7	2

Combining Probabilities - Intersection and Union

- If you bid to make 9 tricks you have to make your 7 definite tricks and both of the possible extra tricks
- Intersection

$P(A \cap B) = P(A) \times P(B)$ if A and B are independent

- P(9 tricks) = P("Finesse succeeds" $\cap$ "3-3 split")

= P("Finesse succeeds") X P ("3-3 split")

= 0.1777

Combining Probabilities - Intersection and Union

- If you bid to make 8 tricks you have to make your 7 definite tricks and 1of the possible extra tricks

- Union

$P(A \cup B) = P(A) + P(B) - p(A \cap B)$

Note if A and B are mutually exclusive $P(A \cap B) = 0$

- P(8 tricks) = P("Finesse succeeds" $\cup$ "3-3 split")

= P("Finesse succeeds") + P("3-3 split")

– (P(Finesse succeeds") X P("3-3 split"))

= 0.6777

6) Posterior Probabilities – Suit-splits

- Probabilities change with each piece of new information
- Bayes Theorem:

$$P(A|B) = \frac{P(B|A) \text{ X } P(A)}{P(B)}$$

- P(3-3 diamond split | Defenders 2 diamonds each)

$$= \frac{\text{P(Defenders 2 diamonds each | 3-3 diamond split) X P(3-3 diamond split)}}{\text{P(Defenders 2 diamonds each)}}$$

- Partition Theorem:

$$P(B) = P(B|A) \text{ X } P(A) + P(B|(\bar{A}) \text{ X } P(\bar{A})$$

Posterior Probabilities – Suit-splits

- P(Defenders 2 diamonds each)

= P(Defenders 2 diamonds each | 3-3 split) X P(3-3 split)

+ P(Defenders 2 diamonds each | 4-2 split) X P(4-2 split)

+ P(Defenders 2 diamonds each | 5-1 split) X P(5-1 split)

+ P(Defenders 2 diamonds each | 6-0 split) X P(6-0 split)

= 1 X 0.3553 + 1 X 0.4845 + 0 X 0.1453 + 0 X 0.0149

= 0.8398

- P(3-3 diamond split | Defenders 2 diamonds each) $= \dfrac{1 \text{ X } 0.3553}{0.8398}$ = 0.4231

Summary

I used:

- Combinations for the:

  Number of Bridge hands

  Odds against a Yarborough

  Prior probabilities of Suit-splits

- Intersection and Union for the:

  Combined probabilities of Suit-splits and Finesse

- Bayes Theorem and Partition Theorem for the:

  Posterior probability of Suit-splits

In my essay I will also use:

- Decision Theory to analyse the Scoring System
- Game Theory to analyse Bridge moves