Some Basic Probabilities - Playing the Odds

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Wikipedia puts it quite succinctly, you need to have some **understanding of probabilities** in order to decide which strategy to adopt in bidding and playing bridge hands. Success is dependent upon the distribution of opponent's cards, and in the absence of other information (evidence) players need to identify the most probable route to success.

Basics of Probability

It is important to understand the basics of probability before looking at them as applied to bridge hands. Here are two little tests.

"I have two children, one of them is a boy. What is the probability that the other is a girl?"
There are three possibilities, girl-boy, boy-girl, and boy-boy (girl-girl is not possible). So the answer is ⅔.
If the question defined the boy as being the first-born, then the possibilities would be just boy-girl and boy-boy, or ½ (50%).
*This is the so-called **Boy or Girl Paradox** which states that of all families with two children and a first-born boy, half of them will have had two boys, but for all the families that had two children one of which was a boy, only ⅓ of them had two boys.*

"I have three boxes A, B, and C. One of them is home to an ♥Ace, and people are asked to find which box contains the ♥Ace"
Let's say someone picks box A, so the probability that they picked the right box is ⅓. Then they are told that box C does not contain the ♥Ace, and they are made an offer. They can keep their original choice, or now change and pick box B. When they originally picked box A they had a probability of ⅔ to make the wrong choice, and nothing has changed, i.e. their box still has a ⅓ chance of containing the ♥Ace. Yet now that box C has been eliminated, box B is now the only alternative. Therefore box B must now have a ⅔ probability to contain the ♥Ace.

In both cases some additional information was provided, firstly the sex of the first-born and in the second example the fact that the ♥Ace was not in box C. So the principle is that in calculating a probability **you must include any additional information provided**.

An important aspect to the way probability theory is applied to bridge playing is the so-called **Bayesian probability**. Wikipedia tells us this is about the way the theory of probability is applied to interpret **reasonable expectations**, e.g. the likely distribution of cards in a bridge hand. The idea is that we can specify a likely distribution of cards (values) in a bridge hand as an ***a priori* probability** and update the probabilities to an ***a posteriori* probability** as new 'evidence' appears in the form of bidding and card plays. At times this can become quite complicated, but even a limited understanding of the basics can really give a bridge player a better insight into the meaning of the bidding and the best strategy to play out the cards in any specific hand.

Examples

Many of the most important playing techniques such as elimination plays, finesses, etc. are based on the way high-card points (HCP), suits and face-values cards are most likely to be distributed. An **example** best highlights this.

Dummy ♠A-10-9 ♥5-4 ♦A-Q-6-5-4 ♣J-5-3

Declarer ♠J-2 ♥A-9-7 ♦7-3-2 ♣A-K-Q-9-6

Here we have a contract of 3NT. Declarer has 8 top tricks, and the lead of ♥2 suggests that ♥ are split 4-4. So declarer can afford to give up the lead once. The finesse of ♦A-Q is a 50-50 chance, whilst the double finesse of ♠A-10-9 (using the Jack) offers a 75% chance that the honours are split and thus the 9th trick will be found.

*To be totally pedantic I have read that you are meant to say declarer **finesses the** Queen, or is **finessing against** the King. What 'finesses the Queen' means is to play the Queen (from Ace-Queen), and is thus the same as saying (writing) the outstanding King is finessed against.*

Here is another simple **example**.

Dummy ♠A-6-4 ♥6 ♦8-7-6-2 ♣K-8-7-5-2

Declarer ♠K-3-2 ♥A-K-7-3-2 ♦A-K-4 ♣A-3

Again declarer is playing 3NT, has 8 top tricks, and the lead was the ♦3 (since dummy holds the ♦2 this suggests a 4-2 split). Does declarer look to the 6-card ♥ or to the 7-card ♣ for the 9th trick? With 7 cards missing in ♥ the most probable split is 4-3 with a 62% probability, and with only 6 missing cards in ♣ the most probable split is 4-2 with a probability of 48.4%. So in both cases declarer must play out the A-K and then play 2 losing rounds in order to find the 9th trick. The article which proposed this example closed by noting that "clearly it is better to play for the 62% in ♥ than for the 48.4% in ♣". But I am not convinced, since what Declarer does not want is a spilt worse than 4-3 in ♥ or worse than 4-2 in ♣. Worse than 4-3 in ♥ is 38%, and worse than 4-2 in ♣ is only 16% (4-2 and 3-3 is fine), so it would be better to play for the drop of ♣ and not ♥. However it is irrelevant even with a 4-2 split in ♣'s declarer does not have the necessary 3 entries to dummy. *This is a good example showing that the reader must really sit back and look at the different hands and truly think about what they mean and what the lesson is.*

In this **example** declarer must decide between a finesse and playing for a split.

Dummy ♠A-Q-8-6-4 ♥J-9-4-2 ♦A-6-5 ♣4

Declarer ♠3 ♥A-K-Q-10-8-6 ♦9-7-3 ♣A-8-2

Declarer is in 6♥ and the lead was the ♦Q. Does declarer play the ♠A-Q finesse for the 12th trick, or attempt to 'establish' the 12th trick by ruffing 3 tricks from ♠A-Q-8-6-4. The finesse is a 50-50 option, but establishing the 5th ♠ depends upon a 4-3 split which has a probability of 62%. Does declarer have the 4 entries to dummy? Yes, ♠A, ♥J, and two ♣ ruffs in that order. In fact this option will also work against a 5-2 split with K-x with one opponent, making the total probability of success 71%.

Now we turn to a slightly more complex **example**.

Dummy ♠4-3-2 ♥K-Q-7 ♦8-3-2 ♣A-J-10-9

Declarer ♠A-K ♥A-4-3 ♦A-Q-6-5-4 ♣5-4-3

Declarer plays 3NT against a lead of the ♠Queen, won by the ♠Ace. Declarer has 7 top tricks and needs 2 additional tricks from ♦ and/or ♣. If Declarer loses the lead, opponents will force out the ♠ King, and may have enough winners to defeat the contract.
From ♦ declarer will need 3 tricks, the finesse (50-50) **and** to make the 3rd winning trick a split 3-2 with opponents (probability 68%), making 3NT+1.
The probability of both these plays working, finesse and a good split, is only ½ x 68% = 34%.
From ♣ declarer will need 3 tricks, taking the finesse (50-50) twice. This requires that the honours ♣ King and ♣ Queen are not both held by the right-hand opponent (and protected ♣K-Q-x). In very simple terms, declarer has a 75% probability to find the ♣ King and ♣ Queen split or nicely placed with the left-hand opponent.
Which is better to play, a 75% probability or a 34% probability?
The question can be made more interesting if declarer holds ♦A-Q-J-5-4. Then declarer no longer needs the finesse **and** the split 3-2, but only needs the finesse **or** a 3-2 split, which will occur with an 84% probability. Now the odds have changed and the best way to play the hand has also changed.

Let's take an even closer look at a slightly different **example**.

Dummy ♠J-10-7 ♥7-4-3 ♦8-4-3 ♣A-Q-6-2

Declarer ♠A-K-Q-6-3 ♥A-J-10 ♦A-Q-5 ♣8-7

Again the contract is 3NT but now the opponents have intelligently lead a ♠ (a good 'safety play', always good to give opponents what you know they will always win anyway).
Declarer can see 8 top winners, and needs just a 9th winner to make the contract. Based upon *a priori* probabilities, declarer has three options. Option A - to use the ♠J and ♠10 as entries to make two successive finesses of ♥A-J-10. This has an *a priori* probability of success of 75% for one of the two finesses. Option B - to make the two different finesses in the minors in order to win one of them, this also has an *a priori* probability of winning a 9th trick of 75% from one of the two finesses. Option C - take one finesse through ♥A-J-10 and then finesse one of the minor suits. The probability to find both ♥K and ♥Q together with the right-hand opponent is *a priori* 25%, and adding that to half the *a priori* probability of Option C (75%/2), adds to 62.5%.
What to do? If the first finesse works, then the problem is solved. But what if the first finesse fails? If the first finesse fails the 'future' probabilities also

change since new information has been provided. You can no longer count on *a priori* probabilities, but must now look at *a posteriori* probabilities, the probabilities change because new evidence has appeared.

Let's take the first finesse of ♥A-J-10 (valid for both Option's A and C) and it fails. In Option C the first finesse of ♥A-J-10 failed, and now taking one of the finesses in the minor suits simply has an *a posteriori* probability of ½ (50-50). But the two finesses in Option A **are linked**, and therefore the *a priori* probability of 75% now drops to an *a posteriori* probability of 67%. So it is clearly better to continue with the original plan to play for a double finesse of ♥A-J-10.
In Option B the two finesses on the two different minor suits are independent, the first finesse failed so the second finesse in the other minor now has an *a posteriori* probability of ½ (50-50).

Conditional Probabilities

What we have seen is that the *a priori* probability is just the ratio of the number of favourable options over the total number of possible options.
Example, in that last hand initially the ♥K and ♥Q in the opponents hands could be split K-Q/x-x, K-x/Q-x, Q-x/K-x, and x-x/K-Q. So each of these options has an *a priori* probability of ¼ (25%). Now you take the finesse and look at the different **'conditional' probabilities**. In the first split K-Q/x-x the King has a 50-50 chance of winning (taken at random from K-Q). For the second split K-x/Q-x, the King should win and we give it a conditional probability of 1, and in the other two splits the King will not win and the conditional probabilities are 0. So the probability of the event occurring is the *a priori* probability multiplied by the conditional probability for each situation. If you do the maths the overall probability of the King appearing with the left-hand opponent on the first finesse is ⅜ (⅛ for the split K-Q/x-x and ¼ for the split K-x/Q-x). Now if the King won the first finesse, what is the *a posteriori* probability of success in taking the next finesse on the Queen? There are now only two options, the split K-Q/x-x and K-x/Q-x. Taking their overall probabilities (⅛ and ¼) and dividing them by the total probability for the King to win the first finesse (⅜) gives an *a posteriori* probability of finding K-Q/x-x (after the King won the first finesse) of ⅓ and an *a posteriori* probability of finding K-x/Q-x (after the King won the first finesse) of ⅔ (67%).

Below we have a short and simple discussion about what are *a priori* probabilities, such as how HCP and suits might be assumed to be distributed **before** any additional evidence is taken into account.
This means that you sit down and pick up your hand, and ask "what are the probabilities (chances) of finding that particular distribution, those particular cards, and that number of points,…". And remember these probabilities are valid for any player picking up any of the 4 hands dealt at the table. Everyone starts with the same chance.
Naturally the bidding and actual playing of cards will change a players assumptions about how the cards and HCP are actually distributed in any specific hand.

Warning - Discussing probabilities is not the same as discussing what might or might not be actually declared and played in competitions, nor is it the same as what might best be played either by humans or by computer programs. In addition, I must stress that we have below what are called *a priori* odds, or what we would expect to be the distribution of HCP, suits or face-value cards in a 'fair', 'random', 'unbiased' world with 'all things being equal'. Often discussions about probability in bridge are actually more about a sequence of **logical deductions** that evolve as the cards are played, i.e. deductions that evolve as the 'odds' evolve with the cards played. What we are discussing on this page are **the probabilities distributions of the cards when dealt**, assuming that the cards were shuffled and dealt properly.

And we must also remember that the distribution of HCP, suits and face-value cards are not the only kind of uncertainty in bridge. Bidding systems are not perfect, and along with the strategies of the opponents (competitive bidding and card play), a different form of uncertainty is introduced which is more dependent upon human psychology than on statistics. It must be said that studies on optimal play almost always focus on understanding the best line of play and defence when all four hands are visible to all participants (i.e. so-called **double dummy**).

A good part of the below discussion on the distribution of HCP and suits is based upon the tables found on some webpages of the Occasional Enthusiast. What I have done is to extract some specific examples from the tables, hopefully these examples are the most interesting for the bridge player.

Point Count

It is possible to calculate the probability to **hold any specific number of high-card points (HCP)** in any specific individual hand. For example you have the highest probability (9.41%) to hold exactly 10 HCP, and a slightly lower probability (9.36%) to hold exactly 9 HCP, but a still lower probability (8.94%) to hold exactly 11 HCP. The probability to hold 10 HCP or less is 56%.

Being dealt **7 to 12 HCP accounts for just over 50% of all hands**. This breaks down to one hand in every four will have 6-8 HCP, two hands in every seven will have 9-11 HCP, and 1 hand in every five will have 12-14 HCP.

Being dealt 15 to 17 HCP accounts for 10.1% of all individual hands (but only about 4% of all hands have the strong No Trump distribution).
Being dealt 16 to 18 HCP only accounts for 5.7% of all individual hands. Moving from a 1NT point count of 16-18 HCP to 15-17 HCP meant moving from 1 hand in 49 to 1 hand in 25, making an opening 1NT twice a likely with 15-17 HCP as with 16-18 HCP.

Being dealt 20+ HCP accounts for only 1.45% of all hands and thus will occur around once in every 70 hands, and being dealt 24+ HCP accounts for only 0.1% (once in every 1,000 hands).
Dropping the strong 2♦'s bid and keeping a strong 2♣ to indicate 20+ HCP is a no-brainer, despite the fact that many experts question the efficiency of the weak 2♦ opening.

A nice way to look at the HCP distribution in a hand is to consider **an evening's bridge playing 24 hands**. You would expect to find no hands with 0-2 HCP, 3 hands with 3-5 HCP, 5 hands with 6-8 HCP, 7 hands with 9-11 HCP, 5 hands with 12-14 HCP, 3 hands with 15-17 HCP, and no hands with more than 17 HCP. Players spend far too much time learning bidding options and conventions for hands with lots of points and don't focus enough on part-score bidding, competitive interventions, and card play. And yes, I know it adds to 23 and not 24, but that just because of the rounding error. You chance of getting a hand with 18+ HCP is still only 4% or 4 times in 100 hands. So maybe you will get one of those hands next week! Good luck, but remember that with at least two of those wonderful 18+ HCP hands, your partner will Pass. Also for the occasional 1NT opening (15-17 HCP), partner will pass half the time (46%) unless you use the Jacoby Transfer Convention (responder has about a 30% chance of having a 5-card major in front of an opening 1 NT).

So given that more than 50% of your 24 hands will only have 5-11 HCP, there is all the more reason to get to know well the weak-two bids even if a hand with a 6+ card suit will probably only occur once for you in an evenings bridge.

Playing with a partner you must add you HCP together. In nearly half (46%) of the 24 hands you will have together 18-23 HCP, so you had better be good at bidding and playing part-scores. Maybe 4-5 hands (23%) will have 24+ HCP and could be played in game. You and your friends, playing 24 hands a week, will have to wait 3 months before getting a small slam with 33+ HCP.

Distribution

The most probable distributions in any individual hand are 4-4-3-2 (21.6%), 5-3-3-2 (15.5%), 5-4-3-1 (12.9%), 5-4-2-2 (10.6%) and 4-3-3-3 (10.5%). These 5 distributions represent more than 70% of all individual bridge hands.

One of the hands that is often quoted a causing problems in bidding systems is the 4-4-4-1, which occurs 3% of the time.

You have a 44.34% probability of holding a hand with one or more 5-card suits, but no 6-card suits.

You have a 35.08% probability of holding a hand with one or more 4-card suits, but no 5-card suits or longer (only the 4-4-4-1 distribution which occurs with a probability of 2.99% is not also included in the NT type distribution).

You have a 39.85% probability to hold an NT type distribution with 4-4-3-2, 4-3-3-3, or 5-3-3-2 with a 5-card minor suit (54% of NT type hands will have the distribution 4-4-3-2, and an NT type distribution with a 5-card minor will occur only 19% of the time).

You have a 16.55% probability of holding a hand with one or more 6-card suits, but no 7-card suits or longer.

You have a 3.53% probability of holding a hand with one 7-card suit, but no 8-card suit or longer.

You have a 0.47% probability of holding a hand with one 8-card suit, but no 9-card suit or longer.

You have a 5.10% probability of holding a void in one or more suits (one in every 20 hands).

You have a 30.6% probability of holding one or more singletons (every 3 or 4 hands).

You have a 53.8% probability of holding one or more doubletons (every second hand).

Points and Distribution

You have a 6.51% probability of holding 12-14 HCP and a 4-3-3-3 or 4-4-3-2 distribution (and an addition 0.54% probability of holding one of those distributions but with exactly 18 HCP). You have a 0.97% probability of holding 12-18 HCP and a distribution 4-4-4-1. You have a 1.59% probability of holding 12-14 HCP and a distribution 5-3-3-2 with a 5-card minor suit. You have a 4% probability of holding 12-18 HCP and a distribution 5-4-2-2 or 5-4-3-1 or 5-4-4-0 with a 5-card minor.
So you have around a 13% probability of holding a hand with between 12-18 HCP and a distribution suited to an opening bid of a 'better minor' (so excluding the strong NT distributions with a 5-card minor suit).

You have a 6.50% probability of holding 12-18 HCP and at least one 5-card major suit.

You have a 4.08% probability of holding 15-17 HCP and an NT distribution (one in every 25 hands).

You have a 2.18% probability of holding a weak 6-card major (6-3-2-2 or 6-3-3-1) with 5-10 HCP, and an additional 1.09% probability of holding a weak 6-card ♦ suit with 5-10 HCP.

Point Count Adjusted for Distribution

Above we saw HCP and card distribution probabilities for a 13-card bridge hand. But what happens if you take that purely statistical distribution of HCP and add additional distributional points based upon the Pavlicek Point Count? Remember this point count system adds 3 points for a void, 2 points for a singleton and 1 point for a doubleton, adjusts for unprotected honours, and adds extras point for long suits, as well as a point for holding all 4 Aces and ½ point for each of the 10's.

What happens is that the probabilities of holding hands with a specific number of total points changes, sometimes quite dramatically.

Based purely on HCP the hand with the highest probability was one holding exactly 10 HCP (9.41%). However adjusted for distributional points for all types of hands holding exactly 10 HCP, the probability of the occurrence of that type of hand drops to 9.02%, and the hand with the highest probability becomes the one with 11 total points (9.16%). Based purely on HCP the probability to hold 10 HCP or less was 56%, now the probability to hold 10 total points or less is just under 44%.

Being dealt 7 to 12 HCP accounted for 51.2% of all hands, but now adjusted for distributional points the range 7-12 total points (inclusive) accounts for only 45% of all hands.

Being dealt 15 to 17 HCP accounted for 10.1% of all individual hands, but now adjusted for distributional points the range 15-17 total points (inclusive) accounts for 18% of all hands.
When looking at the range 15-17 HCP we found that about 4% of hands were No Trump ones. Traditionally we do not count distributional points when evaluating NT hands. However many experts do add ½ points for useful 10's, and some experts down-grade hands with exposed honour cards (e.g. QJ doubleton). The increase from 10% to 18% is largely based upon hands with less that 15 HCP but which have strong distributional characteristics (e.g. singletons, or solid long suits) and thus move into the 15-17 total points range. There are also a few hands that have 15-17 HCP but are re-evaluated at 18+ when distributional points are added.

Being dealt 20+ HCP accounted for only 1.45% of all hands, but now adjusted for distributional points the 20+ total points accounts for 3.51% of all hands.

Partnership Combined Points (2 hands)

The previous probabilities were for individual hands, but it is also possible to calculate combined probabilities for the HCP and card distributions in 2 hands.

Based purely on HCP, a partnership can expect to hold a combined 25+ HCP 17.5% of the time (one in every 6 hands). This excludes additional points added for distribution, vulnerability, etc. And a partnership can expect to hold 30+ points about 2.2% of the time. **Expert players will try to push to game when holding a combined 23-24 HCP, particularly if vulnerable**. A partnership can expect to hold a combined 24+ HCP 23.4% of the time and a combined 23+ HCP 30.3% of the time. Experts consider that they have a fighting chance of game when holding 10 cards in a suit and with a 20-20 HCP split. This means that experts will try for game on up to 50% of their contracts. This equally true for expert opponents, so it can be that up to 50% of all contracts played at a table could be game attempts.

If one partner holds exactly 12 HCP there is a 28.1% probability that the other partner will hold 12 HPC or more. If one partner holds exactly 15 HCP there is a 36.5% probability that the other partner will hold 10 HPC or more, and a 19.1% probability that they hold 12 HPC or more. If one partner has exactly 10 HPC there is only an 8.5% probability that the other partner will hold 16 HPC or more. The probability distributions are not symmetrical.

If a partnership has 25 HCP how will the opponents HCP be distributed? The opponents HCP will be distributed between 5-10 and 10-5 a total of 67% of the time. If the partnership has 26 HCP the opponents HCP will be distributed between 5-9 and 9-5 a total of nearly 60% of the time.

Partnership Combined Distribution (2 hands)

By far the most common distribution across two hands is 8-7-6-5 which occurs with a probability of 23.6% (about every 4th hand). The next most likely distribution for a partnership is 7-7-6-6 with a 10.5% probability. The third most likely distribution is 9-7-6-4 with a probability of 7.3%, and then both 9-6-6-5 and 8-7-7-4 have a probability each of 6.6%. The fact is that if a partnership has an 8-card suit, there is a good chance that opponents will also have an 8-card suit. Or put another way **if your adversaries have bid an 8-card suit, you and your partner also have an 8-card suit**. Of course the distribution of the 8 cards can range from 8-0 through 4-4 to 0-8.

The only distributions that do not include an 8-card suit are 7-7-6-6 and 7-7-7-5, which total only 15.7% of all combined hands. Or put another way, **84% of all distributions across two hands include at least one suit of eight or more cards**. This means that in more than 4 in every 5 games both teams will have at least one 8-card suit.

The definition of a 'good fit' is often used when a partnership holds an 8-card 'primary' suit, and another 7-card or 8-card 'secondary' suit (this will occur about 30% and 10.3% respectively).

One problem that partnership's face is when their two hand have the same shape. It can occur that partnerships play in a suit contract with few ruffing opportunities, whereas they should be playing in NT. There is a 40% probability that if one partner holds 4-3-3-3, then the other partner will hold the same exact distribution. It is nearly the same for two hands having the same distribution 4-4-3-2 (39.6%). The only other significant probability is for two hands with a 5-3-3-2 distribution (12%).

Interestingly if one partner holds a hand with a singleton there is only about a 2.5% probability that the other partner also has a singleton. If partner has a void, then there is only about a 0.03% probability of the other partner also holding a hand with a void.

If one partner holds exactly 5 cards in a suit, the probability of the other partner holding exactly 3 cards in the same suit is about 31% (29% for only 2 cards and only 17% for 4 cards). The probability that the other partner has less than 3 cards in the same suit is 45.6%, so the probability that the partner holds 3 or more cards in the same suit is 54.4%, meaning that **a partnership has a 1 in 2 chance of finding an 8-card fit**.

If one partner has exactly 4 cards in a suit, the probability of the other partner holding exactly 4 cards in the same suit is only 22.2% (or 33.7% for holding at least 4 card support).

If one partner holds exactly 6 cards in a suit, the probability of the other partner holding exactly 2 cards in the same suit is about 33% (28% for 3 cards and only 19.5% for 1 card). The probability that the other partner has less than 2 cards in the same suit is only 23.8%, so there is a 76.2% probability that partner is holding at least 2 cards in the same suit.

Warning - These probabilities are not symmetrical. For example if one partner holds exactly 3 cards in a suit, the probability of the other partner holding 5 cards in the same suit is not 31%, but is only 9.1%. The only symmetrical case is when one partner holds exactly 4 cards in a suit, then the probability of the other partner holding exactly 4 cards in the same suit is always 22.2% whichever way you look at it.

When discussing probabilities players must be careful to understand the exact significance of what is written. If one partner is holding exact 4 cards in a suit there is a 22.2% probability that the other partner is holding exactly 4 cards in the same suit. However there is a 33.7% probability that the other partner is holding 4 or more cards in the same suit. And naturally there is a 44.1% probability that the other partner is holding 3 or less cards in the same suit. You can quickly see the justification for looking for fits of the type 5-3 because with one partner holding 5 cards in a suit there is a 54.4% probability that the other partner is hold 3 or more cards in the same suit.

If one partner has two 4-card suits then there is a 63% probability that they will find an 8-card or better fit in one of the two suits. This rises to 83.5% when one partner has two 5-card suits (for at least one 5-3 fit).

Understanding these probabilities affects directly how partnerships should bid their hands. Imagine a partner with a 6-4 distribution. Is it better to repeat the 6-card suit (and hide the existence of the 4-card suit) or to introduce a new 4-card suit (and hid the existence of the 6th card)? With a 6-card suit there is a 76.3% probability to find an 8-card or better suit. Alternatively showing only a 5-4 distribution there is a 74.9% probability of finding an 8 card or better fit in one of the two suits. So it is marginally better to repeat the 6-card suit. All the more true if the 6-card suit is a major. This is based purely on card distribution and not on game values, etc.

Distribution and HCP for NT Hands

3NT hands can be bid with 24, 25 or 26 HCP, but much depends upon how those points are split between the two hands. With only 24 HCP computer simulations show that a partnership has a 41% probability to make 3NT if the points are distributed 12-12, 13-11, or 14-10, but the probability to make 3NT drops significantly beyond the 18-6 point split (36%). *Another reason for the move to bidding 1NT with 15-17 HCP.*

With a combined 25 HCP the partnership has a 59% of making 3NT with a points distribution of 13-12, 14-11, 15-10, and 16-9, and the probability to make 3NT drops below 50% only for point distributions 21-4 or worse.

With a combined 26 HCP the partnership has a 75% of making 3NT with a points distribution of 14-12, 15-11, 16-10, and the probability to make 3NT drops below 50% only for distributions 25-1 or 25-0. For all distributions of 20-6 points or better the probability of making 3NT still exceeds 70%.

Even or close-to-even HCP splits have the best change of making 3NT. Unbalanced HPC splits down to about 7 HCP in one hand have a 4% lower chance of making 3NT. The logic is simple. Making most 3NT hands will depend upon one or two directed leads (for example to exploit a finesse), but weak dummies provide few or no entires (or useful card combinations in general).

Naturally this discussion depends upon the opponents have their points more or less evenly distributed. NT hands are more difficult to play if the split of points in the hands of the opponents is not more or less even. This is all the more true if the stronger hand of the opponents is sitting behind the stronger playing hand.

We can begin to see why 25 HCP and 15-10, 16-9, and 17-8 card distributions are used as the target for making 3NT (59% of the time), but with just 24 HCP 3NT has only a 41% probability of success. Text books often say that partnerships should always play 3NT with 25 HCP, but 3NT with only 24 HCP is a break-even proposition. The books spend a lot of time looking at "either 8 tricks or 9 tricks" in NT. The problem is that playing 3 NT and making 8 tricks can be a valid risk-reward strategy, but making only 7 tricks can be a 'zero'. How to decide to 'up-grade' a hand and arrive in 3NT with only 24 HCP, and end up only making 7 tricks, or to 'down-grade' a hand and just play 1NT? We know that with 24 HCP, split 14-10, there is a 41% probability of making a 3NT game, a 35% probability of taking only 8 tricks (1-down), and an 18% probability of taking only 7 tricks (2-down). But the reality is that we do not know partners HCP to within 1 point, so often the discussion is a bit academic. Maybe the only sensible way to look at things is to 'down-grade' a hand when non-vulnerable, and to 'up-grade' when vulnerable.

Perhaps a more realistic questions is about already being in 2NT with 24 HCP, and trying to decide to accept or not accept a partners invitation, so bidding 3NT or Passing. Vulnerable it is right to continue from 2NT to 3NT with a better than 32% probability of making game. So with 24 HCP bidding a vulnerable 3NT (over 2NT) is always the best option unless the point split is 20-4 or worse. But with 24 HPC bidding a non-vulnerable 3NT (over 2NT) is profitable only when the point split is 15-9 or less. The conclusion is that with the points split more or less evenly (15-9 or better) always accept an invitation and bid 3NT. This solves the "1 NT - 2 NT - ?" question. The same computer simulations show that it is not profitable to routinely bid 3NT with only 23 HCP, vulnerable to not.

Probabilities look a little academic and dry, but they have a direct impact on both bidding and card play. A common sequence of bids might be 1♦ - 1♠ - 2NT, showing that the opener has a balanced 18-19 points. If responder replied with an honest 6 HCP they should always bid a vulnerable 3NT (i.e. they have a greater than 32% probability of making a vulnerable game). With an honest 6 HPC but a non-vulnerable game they should Pass (in this situation partner needs at least 7 HCP to make a game bid worthwhile). A partner that replied with a shoddy 5 HPC has pushed opener into a 2NT bid with only a 36% probability of making just 2NT.

In the above example just 1 HCP can make a difference. In fact in part-score and game contracts 1 HCP adds ½ a trick to the probability of success (this advantage disappears in slams). For example KQJ is worth 2 tricks, whereas AQJ is worth 2½ tricks if entries exist for the finesse.

Based purely upon probabilities it would be wise to pass a 20-21 point 2NT bid when holding 4 points or less. However distribution is not the only criteria. Access to the weak hand could be a determining factor in being able to make a finesse or to developing a long suit. So a partner might pass with Qxx, Jxx, Jxx, xxxx, but bid 3NT with Axx, xxx, xxx, xxxx or QJxx, xxx, Jxx, xxx.

A word of warning, computer simulations are about simulating the different distribution of cards (and thus points) but they assume that the games are played in the best way possible against the best defence possible. So you better know how to best play the hands. Playing for 3NT with only 24 HCP could be a waste of time if you do not know how best to play the contract. However playing 3NT with only 24 HPC could be easily if the opponents are not good players and make the wrong lead. You decide.

But before you decide, consider the results of this study of alternative bidding and play options on a very large set of double dummy hands. What they found was that there was virtually no chance of 'going down' in the game contracts that were actually bid and where the partnerships held 26+ HCP and a least 8 trumps. What was surprising was that 77.1% of the contracts that could have been game declared and 'made' were in fact not bid. Clearly real world bridge players tend to prefer a system that keeps the 'going down' rate low, and are less concerned about missing game contracts. The problem is to find the bidding system that actually predicts success better than using a simple HCP model. Some experts have suggested that club players do not properly integrate distributional points into their HCP bidding models.

Probability of Holding Specific Cards

A player has a 51.8% probability of holding 2 Aces when they have 16 or more HCP.
A player has a 55% probability that a strong NT hand (15-17 HCP) is holding 2 Aces (1.6% holding 0 Aces, 26% holding only 1 Ace, 17% holding 3 Aces, and less than 1% holding 4 Aces).
A player has a 50% probability that a 2 NT hand (20-21 HCP) is holding 3 Aces (7% probability that the hand had all 4 Aces).

Missing Card Splits (in opponents hands)

2 missing cards will split **1-1 52% of the time** and 2-0 48% of the time
Why 52% and not 50-50 or even ⅓? With dummy a partnership holds a specific set of 26 cards, and there are 10,400,600 different ways the remaining cards can be distributed with the opponents (of course each holding 13 cards). There are three ways 2 cards can be split, one or other opponent can hold both cards, or they can be split 1-1. Looking at one opponent (the other opponent automatically gets whats left) they either are holding the two cards from a possible two (and 11 others taken from the remaining 24), or one card from a possible two (and 12 others taken from the remaining 24), or no cards from a possible two (and all 13 cards are taken from the remaining 24). The number of combinations for these three options are 5,408,312 for the spilt 1-1, and logically enough 2,496,144 for each of the other two options (2-0 and 0-2). So this means that 5,408,312/ 10,400,600=0.52 or 52%.

3 missing cards will split **2-1 78% of the time** and 3-0 22% of the time

4 missing cards will split **3-1 49.7% of the time**, 2-2 40.7% of the time and 4-0 9.6% of the time
One of the 'classical' card playing challenges is when declarer holds 9 cards in a suit with the two top honours, but is missing the Queen. The question is to finesse or to play Ace and King to drop Q-x? A simplistic reply might be that a 2-2 split only occurs about 40% of the time, so a player must finesse. We must remember that a 3-1 split occurs with a probability of 49.74% and a 4-0 split occurs with a probability of 9.56%. Often the finesse can be taken only in one direction, but occasional it can be taken in either direction. In the first case the exposed honour should be played. There is a 12.4% probability that the Q is a singleton in a 3-1 split. There is also a 4.78% probability that the Q is still on the right side (for declarer) in a 4-0 split. So the total probability of dropping a Q by playing a top honour and adapting to the outcome (play a successful finesse or play to drop) is in fact 58%. Naturally this is based purely on the probabilistic distribution of cards, and often more information can be inferred from the bidding and play of the cards.

5 missing cards will split **3-2 67.8% of the time**, 4-1 28.3% of the time and 5-0 3.9% of the time
If declarer is holding with dummy 8 cards in a suit and Ace and King, they have only a 27% probability of dropping the Queen as a singleton or doubleton, so a 50-50 finesse is better odds, and they can be improved to 55.7% by playing out one top honour before taking the finesse (i.e. the Queen could be a singleton).

6 missing cards will split **4-2 48.4% of the time**, 3-3 35.5% of the time, 5-1 14.5% of the time, and 6-0 only 1.5% of the time.
If declarer holds 7 cards in a suit, how best to play the odds? Holding A-K-Q-10 and x-x-x in dummy declarer might need to finesse the Jack. If declarer looks to break the suit first (two rounds), then tries for the finesse, they increase the probability of success from 50-50 to 67.5%.
If declarer holds A-J-10-x with x-x-x in dummy, there are only 4 relevant distributions of honours, both on the left, both on the right, and 2 ways for them to be split. By taking the finesse twice declarer has a 75% probability of making 3 out of the 4 tricks.
If declarer is holding 7 cards in one suit and 8 cards in another (but with the Queen missing), the split 3-3 in the 7 cards suit has a probability of 35% whereas the probability to drop a Q doubleton is only 27%. The probability to finesse the Queen is 50-50.

Will Opponent's Ruff?

The probability that opponents will ruff (trump a trick) depends upon the number of cards left in their hands. For example, if only 2 cards are held by opponents there is a 48% probability that one opponent is holding both, and therefore the other opponent has a void (and could ruff on the 1st trick played in that suit).

When opponents are holding 3 cards in a suit there is a 22% probability that one opponent will ruff on the 1st trick played in that suit, and a 100% probability that they ruff the 2nd trick played in that suit.

When opponents are holding 4 cards in a suit there is a 10% probability that one opponent will ruff on the 1st trick played in that suit, and a 60% probability that they ruff the 2nd trick played in that suit.

When opponents are holding 5 cards in a suit there is a 4% probability that one opponent will ruff on the 1st trick played in that suit, and a 32% probability that they ruff the 2nd trick played in that suit.

When opponents are holding 6 cards in a suit there is a 2% probability that one opponent will ruff on the 1st trick played in that suit, and a 17% probability that they ruff the 2nd trick played in that suit, and a 65% probability that they ruff the 3rd trick played in that suit.

When opponents are holding 7 cards in a suit there is a 1% probability that one opponent will ruff on the 1st trick played in that suit, and a 8% probability that they ruff the 2nd trick played in that suit, and a 38% probability that they ruff the 3rd trick played in that suit.

Playing Tricks

The expression 'playing tricks' often pops up in bridge manuals and is particularly useful when looking at 6+ card suits for weak-two and pre-emptive bids. You count 1 point for each winning card (e.g. A-K-Q would be 3 points), and 1 point for the 4th, 5th, 6th, cards, etc. You also count ½ points to represent potential winning honours (e.g. K-x would be ½ point, K-J-10 would be 1½ points, A-Q-10 would be 2 points, and A-Q-J would be 2½ points).

The most probable are hands with 5 playing tricks (12.6%). Around 45% of all hands hold between 4 and 6 playing tricks.

Losing Tricks

You will remember that the losing-trick count is useful for evaluating hands that could be played in a suit. Each suit can only have 3 losers. A singleton is just 1 loser, an A-K-x-x would also be just 1 loser, and a J-10-x-x-x would be just 3 losers. Generally an opening bid is assumed to have 7 losers, and a reply at the one level is presumed to represent 9 losers. Subtracting the number of losers for the partnership from 18 tells us the level that the hands can be played at, e.g. 18 - 7 (opening bid) - 9 (one level reply) = 2, suggesting that 8 tricks can be won and a 2-level contract would be appropriate.

The most probable number of losers in a hand is 8 (24.5% of all hands hold 8 losers). Holding 7 losers has a probability of 23%, or about 1 hand in 4.

Holding 10 losers or more happens 10.4% of the time, or 1 in 10 hands should be automatically Passed unless you are forced to speak.

Holding exactly 9 losers happens 17.6% of the time, or around 1 in every 6 hands.

Holding only 5 losers happens only 6.9% of the time, and holding 4 or less losers in one hand happens just 2.8% of the time.

Quick Tricks

Quick tricks are also often called 'defensive tricks' and represent tricks that might be won taken from just the two top cards in each suit, and assuming that the shortest suit in the hand is trumps. A-K-J-10 would be just 2 quick tricks, A-Q-J would be 1½ quick tricks, K-Q-J-10 would be just 1 quick trick, and even a singleton K would be a ½ quick trick.

About 50% of all hands have 1, 1½ or 2 quick tricks. Nearly 70% of all hands have 2 or less quick tricks, and only 20% of hands have 2½ or 3 quick tricks in them.

Some players expect an opening hand to hold at least 2½ quick tricks, and that game is possible with a combined 5 quick tricks. Just as with losing tricks, quick tricks can be used to evaluate 'boarder line' hands. You must/will always open the bidding with 14 HCP, but do you open with 12 HCP or 13 HCP and only 1½ or 2 quick tricks? Using quick tricks can stop players adding too many 'soft values' to their hand, and can certainly help in deciding to compete, to double, or to Pass. Milton Work attributed 4 points for an Ace and 3 points for a King to stress how important these cards are in controlling the timing needed to make game. Opponents who hold Aces and Kings can gain time to create addition winners, and defeat a contract. Some players can go too far in the bidding when they try to offset the lack of quick tricks with 'soft values', and this is also true for those players that double a game contract based on a soft HCP count rather than on quick winners. Example, holding Q-J-x, J-x, A-x-x, Q-J-10-x-x, looks nice as a defensive hand, but actually only holds 1 quick trick. Keeping the same number of points, this hand K-Q-x, x-x, A-Q-x, x-x-x-x-x has 2½ quick tricks, and could be even stronger if sitting behind the opener.
Some players judge their competitive overcalls and defensive bids based upon quick tricks.

It is very important to only use a 'take-out' Double in a competitive auction when holding good quick tricks. This kind of Double in a competitive auction informs partner about the **nature** of your HCP holding, e.g. it could mean 13 HCP and with at least 2 quick tricks. This helps a partnership understand when to Pass or Double the final contract. If partner is holding 13 HCP in 'softer' values an overcall might be a better option than making a defensive oriented Double.

Many expert players have said that quick tricks and controls are the key to make a good assessment of a bridge hand.

Controls

We saw that Aces and Kings are controls, and controls can be easily counted if you give Aces 2 points and Kings 1 point. Hands are most likely to hold 3 controls (20.9%) or 2 controls (20.5%). More than 60% of all hands contain 3 or less controls, and nearly 80% have no more than 4 controls.

Taking a Finesse

The finesse is one of the basic card playing techniques, but as we have seen on this webpage the finesse is essentially a question of probabilities (even if the bidding and card play can be invaluable in helping declarer find the best line of play).

Declarer has an *a priori* probability of 50% of success with a finesse in the following situations:
A-Q-x in front of x-x-x there is a 50% probability of winning 2 tricks
Q-J-10 in front of A-x-x there is a 50% probability of winning all 3 tricks
Q-x-x in front of A-x-x there is a 50% probability of winning 2 tricks

Declarer has an *a priori* probability of 25% to take two different successful finesses
Declarer has an *a priori* probability of 12.5% to take three different successful finesses

Declarer has an *a priori* probability of 75% to take two different finesses, one of which will be successful
Declarer has an *a priori* probability of 50% to take three different finesses, two of which will be successful
Declarer has an *a priori* probability of 87.5% to take three different finesses, one of which will be successful

Lets consider two different hands A-Q-10 and A-J-10, the probability to successfully finesse both honours is 25%, the probability to win one of the finesse and make two tricks out of three is 50% when the honours are split, and the probability to lose both finesses is 25%. Therefore the *a priori* probability to win two tricks by finessing both honours is 75%, but if declarer loses the first finesse, the second finesse has a 67% *a posteriori* probability of success.

Declarer should play these finesses are follows:

A-Q-J-10-x-x in front of x-x-x-x-x, with 11 cards play the Ace for a 1-1 split (52%) and drop the King

A-Q-J-x-x in front of x-x-x-x-x, with 10 cards finesse because playing the Ace to drop the King has only a 26% probability

K-x-x-x in front of A-J-x-x-x, with 9 cards play King and then finesse the Queen has a 47.8% probability, against a 52.2% to drop the Queen by playing Ace and King

A-x-x in front of K-J-x-x-x, with 8 cards play Ace and then finesse the Queen has a 52.8%, against only 34.7% to drop the Queen by playing Ace and King

K-J-10-9 in front of A-8-x-x, can be finessed in either direction each with a 50% probability, but the opponents bidding and play may provided additional information

A-K-10-7 in front of Q-x-x, play Ace then Queen, keeping open the possibility to finesse the Jack, but without any additional information there is slight advantage to playing the King to drop the Jack

A-K-Q-10 in front of x-x, play Ace or King then finesse the Jack is much better than hoping to drop the Jack

A-K-Q-10-x in front of x-x, there is a small advantage to playing out Ace, King and Queen hoping to drop the Jack

K-Q-10-x-x in front of A-x, there is a small advantage to playing out Ace, King and Queen hoping to drop the Jack

These 'basic' forms of finesse can be summarised as:
Finesse is always better if you have 10 cards and are missing the King, or with 8 cards and are missing the Queen, or with 6 cards and are missing the Jack.
It is ***marginally*** better to go for the drop if you have 11 cards and are missing the King, or with 9 card and are missing the Queen, or with 7 cards and are missing only the Jack (but have the 10).

There are hand where a finesse is needed, but the question is how to finesse or what to finesse.

Holding J-8-x-x-x with A-Q-9-x-x in dummy many declarers would look to a simple finesse of the King by playing up to the A-Q pair, but the better option is to play J which also protects against K-10-x being held by the left-hand opponent.

With almost the same hands as above, but holding J-8-x-x with A-Q-9-x-x in dummy the better approach is to finesse to the A-Q. Then if the 10 drops from the right-hand opponent, declarer can finesse the King again, and if the right-hand opponent drops a small card, then declarer plays the Ace hoping for a split of K-x/J-x. If declarer plays the Jack and the left-hand opponent covers it with their singleton King, the declarer will lose a trick to 10-x-x held by the right-hand opponent.

Holding J-8-x-x with A-Q-9-x in dummy the better option is to finesse with the Jack. If the Jack is covered, declarer can finesse the 10 against Q-9. This is the best option, even if the chance of success is limited (approx. 27%).

Holding x-x-x-x with A-J-10-x-x in dummy first finesse through the 10, and if it loses finesse again through the Jack. This has a 75% probability of winning one of the two finesses.

Holding K-x-x-x with A-10-x-x-x in dummy (note the missing touching honours) it is best to plan to play Ace and King for the 2-2 split (52%). Why is it better to first play the King? What happens if declarer plays the King and the right-hand opponent drops one of the two honours (Queen or Jack). New information has been provided which changes the *a posteriori* probabilities. Now the probability to drop the other honour by playing the Ace is only 35.3%, and the probability to finesse the outstanding honour through A-10 is 64.7%.
Let us think about this. Before playing the King there were a large number of ways that 4 cards could be distributed across the opponents hands (including Q-J-x-x/- through to -/Q-J-x-x) and a small advantage for the split 2-2. Now with one honour from the right-hand opponent falling under the King there are only three possible explanations. The options are now Q-x-x/J, J-x-x/Q and x-x/Q-J and two of those three options require a finesse.

Holding K-x-x-x with A-10-9-x in dummy, first play the King then finesse twice irrespective of the card that drops from the right-hand opponent.

Holding x-x-x-x with A-Q-10-x-x in dummy finesse through A-Q and if it loses play out the Ace to try and drop the Jack.

Holding one card less with x-x-x with A-Q-10-x-x in dummy finesse first though the Q-10 and if it loses to the Jack finesse again through A-Q for the King.

Holding x-x-x-x with A-Q-J-x-x in dummy the *a priori* probability of the finesse is 50-50 (50%). But declarer now plays x to the A-Q, and the left-hand opponent plays a small value card x. Does the probability of a finesse change? The fact that the left-hand opponent has played a card provides more evidence concerning the possible distribution of the remaining cards, i.e. we must now look at *a posteriori* probabilities. For example it eliminates the distribution -/K-x-x-x. If we focus on determining the relative probabilities of a successful finesse versus successfully dropping the King by playing the Ace. Because we are looking the relative probabilities of success we ignore all the distributions where the finesse fails with the King protected with the left-hand opponent. The only distribution that would successfully drop the King is x-x-x/K and this has an *a posteriori* probability of 13.75%, the remaining 86.25% tells us that we should continue as planned to take the finesse and to ignore the fact that the left-hand opponent has played a low value card.

"With 8 ever, with 9 never"

This so-called 'rule of thumb' is based on an understanding of probability, or is it? I've taken this description from the same person who wrote up the example just above. What "with 8 ever, with 9 never" just means that a player should always finesse a Queen if they are holding an 8-card suit, but never finesse a Queen with a 9-card suit.
Let's consider first the 8-card suit. Opponents hold 5 cards, split 3-2 (or 2-3) with a probability of 2x0.3391 = 0.6782 (67.8%), split 4-1 (and 1-4) with a probability of 2x0.1413 = 0.2826 (28.3%), and split 5-0 (and 0-5) with a probability of 2x0.0196 = 0.0392 (3.9%). Declarer will need entries to take a finesse and should play a top honour (Ace or King) first before finessing the Queen. Playing out a top honour is a protection against one of 5-0 or 0-5 splits, since the finesse can then be made against a known distribution (probability of finding the Queen on a 5-0 split on the 'good' side is 0.0196 or 1.96%). Playing out a top honour also protects against a split of 4-1 (or 1-4) where the Queen is a singleton (probability of 0.0283 or 2.83%). Playing out a second card for the finesse will drop the Queen in a doubleton (distribution 3-2) on the 'good' side (probability 0.1357 or 13.57%). The probability that the finesse is a success (against a distribution 3-2 with the Q-x-x on the 'good' side) is 0.2035 or 20.35%. And finally the probability that the finesse is a success (against a distribution 4-1 with the Q-x-x-x siting on the 'good' side) is 0.1413 or 14.13%. Adding together all the 'positive' probabilities makes 0.0196 + 0.0283 + 0.1357 + 0.2035 + 0.1413 = 0.5283 or 52.8%.
A 52.8% probability of making a successful finesse might not appear that interesting, but it must be compared with the alternative. Holding an 8-card suit and playing Ace and King will drop the Queen with a 34.7% probability. So which odds do you prefer 34.7% or 52.8%?

So thats the "with 8 ever", but what about "with 9 never"? This time declarer has with dummy a total of 9 cards in a suit, but is still missing the Queen. The odds are 40.7% for a split 2-2, 49.7% for a split 3-1 (and 1-3), and 9.6% for a split 4-0 (and 0-4). The routine is the same, take a first trick with one of the top honours, then finesse. Declarer will have a probability of 56.2% for the finesse, but will have a 57.9% probability of dropping a Queen singleton or doubleton (and including a 'marked' on the 'good' side). The reason why many experts warn against a simplistic "with 9 never" is that the odds are not so different, and there could be good reasons to still take the finesse. For example, with just a 1.7% advantage for the split, declarer might decide that it is more important to play the finesse so that a dangerous opponent does not gain the lead.

More information

If you want to delve into the finer details of these probabilities check out the Occasional Enthusiast and TaigaBridge. You may have to hunt around a bit but all the above data can be found 'somewhere' on the web, or you can also calculate them yourselves. Here are a few more webpages that might be of interest, Hand Evaluation Stats, Durango Bill, Tommy's Bridge Blog (last updated Aug. 2015), Lessons with the Texas Capital Bridge Association, For Bridge Players (and the webpage Odds & Theory), Richard Pavlicek, and Bridge Winners and a whole series of pages on Conditional Probabilities and Bayes' Theorem.
Many of the statistical tables on these different websites are the same, but not all, so it's important to read carefully the exact description of the

calculations being made.

Sick of Probabilities?

So many percentages and probabilities, it's impossible to remember them all, so you end not remembering any of them. The reality is that better players know the odds, but the importance is not the exact percentages, but to know the better alternatives. So which one are worth remembering?

Here I've copied out **what people think** are the most important ones (it does not mean they are right).

"The odds to end up playing 1♣ are 5/10,000, so don't be afraid of opening 1♣ with a short suit"

"The most frequently bid contract at the 1-level is 1NT, at the 2-level is 2♠, and at the 3-level its 3♠" (note: - 'bid' does not mean 'made')

"You have more than a 70% chance of bidding and making 1♥ or 1♠"

"The most frequently played 'game' contract is 3NT, followed by 4♠ and then 4♥"

"The most frequent contracts where players miss 'game' is when they stop in 2NT, and then in 2♥ or 2♠"

"Contracts of 3♥ or 3♠ go down more often than you think, in part because of poor invitational bids and in part due to competition for the contract".

"An opening 2NT followed by three Passes, goes down 60% of the time"

"3NT makes more often than it should, in part because it includes unbid 4NT and 5NT hands, whereas the least frequently played contract is 5NT"

"Too few minor contract slams are bid and made, in part because too often players stop in 3NT"

"85% of 7NT contracts are made"

"The average number of tricks made is 9, and there is a 42% chance of making 9 or 10 tricks".

- And: -

"If the number of cards you and dummy hold are an even number, the probability is that the odd number of cards held by the opponents will split as evenly as possible, The probability of this division is always more than 50% and gets greater as your holding progresses from 6-8-10 cards. Remember the Rule, forget the specifics"

"If the number of cards you and dummy hold are an odd number, the probability is that the cards in the suit held by opponents will split unevenly. The probability of a even split increases as your holding progress from 5-7-9 cards, but never reaches 50%"

This is the same as saying.
"An even number of cards probably don't split evenly" - i.e. 6 card missing will probably split 4-2
"An odd number of card probably do split evenly" - i.e. 7 cards missing will probably split 4-3
"But when holding 11 cards in a suit play for the split 1-1 (52%)"

Next: -

"If you hold 9 or 10 cards you should finesse a King, but not a Queen or Jack"

"If you hold 7 or 8 cards you should finesse a King or Queen, but not a Jack"

"Two finesses will both succeed only 25% of the time", but "one of two finesses will succeed 75% of the time"

"If one of your opponents bid or doubled, they are most likely to be holding the missing Ace or King"

"If one opponent is long (or short) in one suit, they are likely to be long in another suit"

"The opponent who holds a long suit is most likely to hold the missing honour card in that suit"

"If one opponent drops one card of a missing pair, then the odds are they don't have the other missing card"

Sometimes declarer is faced with taking either a finesse (50-50) or going for a split, but often they can't remember the probabilities. What is really important is only to remember those splits which have a **probability of success greater than 50% or are the most probable**. In fact 4 cards split 3-1 with a 50% probability, 6 cards split 4-2 with 48% probability and 8 cards split 5-3 with 47% probability. All the other splits 1-1, 2-1, 3-2, and 4-3 have better odds than a 50% finesse.

If you hold a 5-card suit there is a 54% chance that your partner is hold 3 cards in the same suit, and if you have a 6-card suit then there is a 75% chance that you partner is hold 2 cards in the same suit. But if you have only a 4-card suit, then there is is only a 33% chance that your partner is also holding 4 cards in the same suit.

http://bernardsmith.eu/pastimes/bridge/basic_probabilities/