Dropping the King Offside with a Ten Card Suit!

(A.K.A. should I have paid more, or less, attention in High School?)

A player came to me at the end of an evening and told me about a hand where he, sitting West, had South as Declarer, drop his King with a Ten card suit across both hands (six on the table & four in hand). He had stated that Declarer started from the table, and when partner played low, Declarer hopped the Ace and his King fell. West stated that he thought South should have run Dummy's Queen into West's King and that, that is how he would have played because he believed that *"finessing"* sic was the 50% play. My conversant went on to say that, South (on completion of the hand) had stated, from information he'd learned on *"The Internet"*, that he should play for the King to drop.

Now the dreaded question… I was asked; *"What are the chances of the King dropping in that situation?"*

Well… It was the end of the evening. My partner was driving that night, and was pretty keen to be heading home. Though, I was happy to quickly volunteer upon seeing the Hand Record that *I would have let some high card run* into West's King, and that I was reasonably sure that running the Club Queen/Ten into West's hand was probably the right play. Needless to say, my (slightly tentative) advice wasn't met with the highest level of outright acceptance. The response was something along the line of *"I thought you might have had some sort of table or information that could tell me what the odds are?"* and I did have a table in a document in my mobile phone somewhere (which I was carrying), but I wasn't entirely convinced that the table I had would quite answer that particular question, and I didn't really want to start fossicking around in my phone/organiser to find it. Truth is, I don't have a photographic knowledge of the contents of every bridge document in my organiser, and I've even less of an idea of where the thing chooses to put some of the documents versus some of the others. (I'm starting to think there might be a random number generator involved somehow!)

The Hand

	♠ 8 ♥ A ♦ AJ976 ♣ QT9432	
♠ J653 ♥ Q76542 ♦ 53 ♣ K	N W E S	♠ KQ97 ♥ JT98 ♦ Q84 ♣ 76
	♠ AT42 ♥ K3 ♦ KT2 ♣ AJ85	

Bd. 13, North, All

South was in 6N and I'm guessing by the fact he made twelve tricks including the fact that he'd managed to drop the Club King means; there was a failed Diamond finesse at some point and very probably a Heart opening lead. I'm not actually interested in discussing the hand beyond setting the context of an opening heart lead by West to Dummy's Ace followed by an immediate run of a top club, at which point East follows with a choice of low equals; or in the context of Declarer at the time, East follows low in tempo. (Though I should indulge the double-dummy experts by acknowledging there is a pseudo Spade-Diamond squeeze on East by Declarer's ducking a natural opening spade lead for one round, given that we know Declarer will succeed in dropping the Club King on this particular occasion).

Please wait… Performing self-check…

I was pretty confident in my assessment of the hand, and my advice, *that running a high card from the table* was the right line of play. But there was still that nagging question about the likelihood of dropping the King offside, reinforced somewhat by the fact that it actually worked on this particular occasion. *I was a bit less convinced about the expert advice gleaned from online.* So I consulted my oracle (and a few tables).

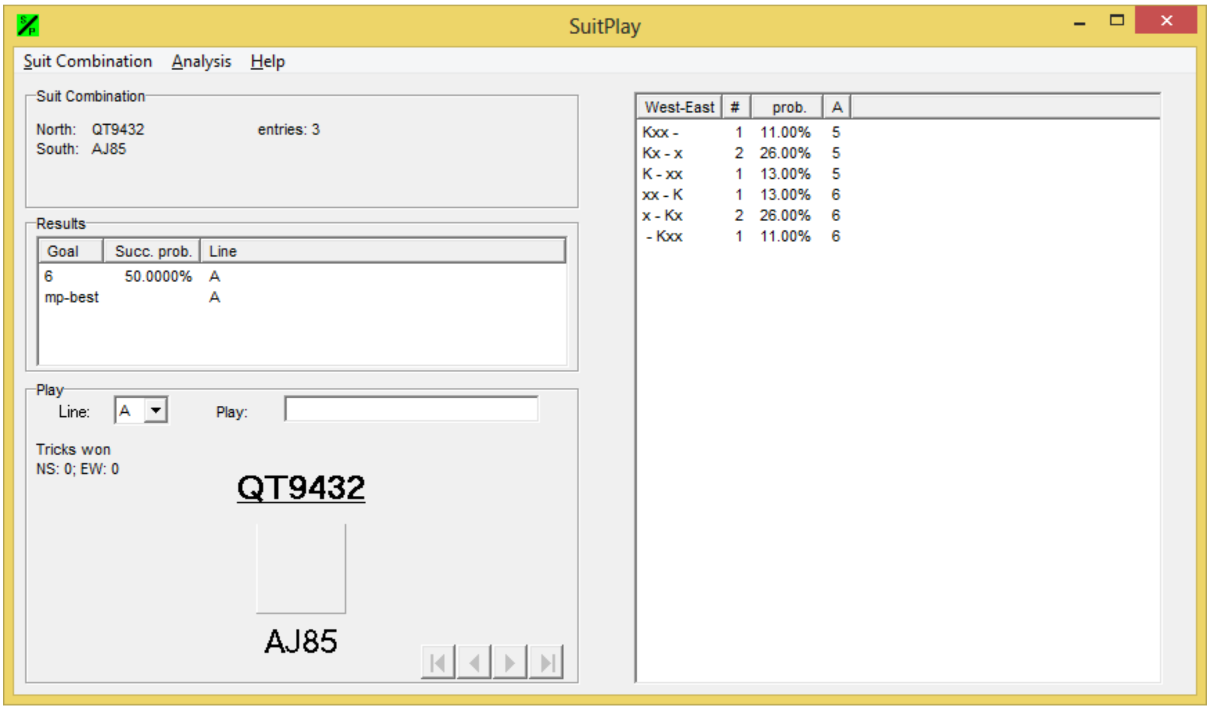

Fig. 1

Okay, so there is only one line with this combination, and it is?

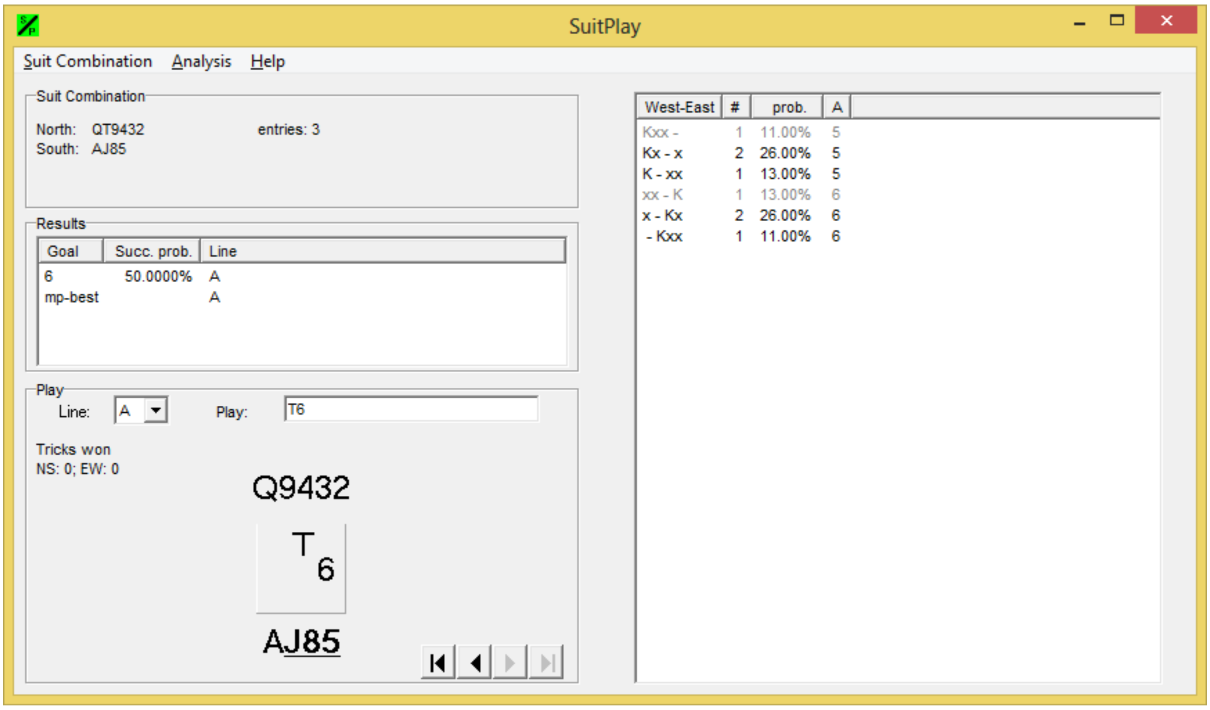

Fig. 2

Running the Queen/Ten, or finessing!

Phew! I wasn't going crazy *and I haven't lost the plot!* Additionally, checking the Fifth Edition of "The Official Encyclopedia of Bridge" (sic) showed precisely the same single line of play; "Run the queen. Don't finesse the jack in case West is void." *Running the queen removes the need to re-enter dummy for a second finesse (for the less experienced players out there).* For clarification, (see figures 1 & 2)

SuitPlay will only allow the play of <u>underlined</u> cards. After the selection of the Ten followed by East's Six, I am not given the choice of selecting the Ace. There is only one selectable line of play (Line A).

Incidentally, for anyone who hasn't discovered "SuitPlay", it's a nifty little program I discovered about a decade ago, and the latest version will take into account;

- The number of available entries
- Discards/Ruffs
- The number of vacant places in each defender's hand (alters the probability for each case, re *Principle of Restricted Choice).*
- Will show the best line available for either IMPs, or Match-Point play.

It can be found by searching for 'suitplay' on the Net, **and its freeware!** It's an easy way for me to either check that I've been following the correct line, or that I have the right ideas when I'm practising at home.

But you haven't answered the question yet!

At this point I wasn't quite sure if I'd gotten to the bottom of my question, which may have become more an issue for me than it was for my original answer seeker. Maybe he'd have happily accepted my findings (that running the Queen was the right play) and moved on. I don't entirely know how much West understood of the question he was asking. Maybe he had a full appreciation of prior and posterior probabilities, maybe not? The truth being that this was immaterial, *because I believe it is up to me to answer the question fully,* simply due to the fact that *I believe I did have an understanding* of the subtext in the question. To refresh our memories, the question was;

"What are the chances of the King dropping <u>in that situation?</u>" It was this last part of the question that had me over a barrel in the first place, and was exactly the reason my answer on the night betrayed itself as being somewhat tentative. My understanding of the question was… **What are the odds of dropping the singleton King offside when running an Honour *and East follows <u>Low</u>?***

Now, I'm not entirely sure everyone in reader land is still on the same page as me, or indeed, still on the same page full stop! For those of you who are still with me (and might not have previously realised), the odds we get from a table in a document or text usually only relates to *prior* odds. These probabilities are classically referred to as "A priori" probabilities. Now there are a whole other bunch of probabilities out there that we don't always like to concern ourselves with, those being the trickier bunch of odds, called *posterior* odds, or "A posteriori" probabilities. The big issue with this bunch of odds is that they keep changing every time we get new information. Like East following *Low* when we run the Queen with a ten card suit, or as the experts like to refer to it, *playing a choice of equals.* Once this has happened all the odds change. Sometimes they change a little and sometimes they change a lot. Take our above hand under a different scenario as an example. Let's say that after Declarer calls for the Queen from the table, instead of playing a low Club, East discards, showing out. We now know that (notwithstanding either opponents having dropped, misplaced, secreted on their person, ingested, or due to any other Bridge-player specific quantum anomalies) West has a 100% certainty of holding Kxx in their hand. The prior probability of 11% becomes a posterior probability of 100% that West is holding Kxx in Clubs.

The crux of my issue was this. I didn't believe I'd fully answered West's question until I could give a definitive answer *knowing* that East had followed low to Declarer's call for the Queen.

The Principle of Restricted Choice (for the rest of us…)

I'm sure we've all had it at some time. I know I have, and possibly I've been guilty of it on maybe an occasion or two in my earlier years of competition. It's the end of a hand and you've somehow ended up with an outright bottom due to some uncanny Card Play, and our 'Declarer Opponent' casually, or even flamboyantly, mutters something to partner about the correct play being due to the (somewhat heinous at this moment as far as we're concerned) "Principle of Restricted Choice" and they both have a smug little chuckle. It's usually accompanied by some comment along the lines of, "It seemed like a pretty obvious play to me…" As if somehow asserting that this opponent knows some special secret about the game that the rest of just don't get?

So what's it all about? Well, to put it into its most basic of explanations for the rest of us, the Principle of Restricted choice pretty much relates to; *the person to my right, or left, just played that card (whatever it may be) because they had to.*

To expand on it, your opponent may have played a particular card, because in relation to the rest of the hand (or perhaps their own hand), it just didn't matter as much as the others, or (sometimes) it was their least of worse choices.

As far as I'm concerned, until you're particularly interested in developing this specific area of your game, or you've just had an emergency phone call imploring you to go and play for your country at some international event, this is about as much knowledge as many of us may need for the significant part of our Club Bridge career.

Enough ranting and back to topic, now that we all have a basic grasp of this particular principle.

The High School bit. (Bayes' Theorem)

Here was I, with an issue I knew could be resolved, but not enough of a realisation what I might have to do to resolve it, nor did I have a good enough recollection of prior years' education to discern what the new probabilities would become once East had followed low to the Club Queen lead. I had an idea of what the posterior probability was, and I thought I could work it out from the remaining probabilities in the table of Figure 2, but how was I to prove it for myself, and more importantly explain it with conviction to the poor soul who'd seemingly asked me such a simple question in the first place? Not to mention the possibility of ending up in a potentially confrontational conversation with a Declarer, whom had *so successfully* dropped the stiff King in the first place?

Well, of course my first option was the old faithful tried and tested technique. I'd look it up on the Internet, like so many other bridge facts I've come to discover over the years. I looked, and failed, which struck me as odd… Generally I've found something that explains almost everything when I search online, but nowhere could I find a table that related to a posterior probability for dropping the King on the left after a low card on the right when holding a ten card suit. Surely it couldn't be that hard to discern an accurate and absolute answer to such a problem?

I did manage to find a whole lot about card distribution probabilities and some such, with multiple versions of the same information, but not what I was after. The closest I could come to it was a good old Wikipedia article about conditional probabilities, which did have this strange mathematical method of working out pretty much what I needed to know. Bayes' Theorem!

"Now where had I seen this before? Hmm…" it dawned on me, my only last and vague recollection of Bayes' Theorem (to the best of my knowledge) involved a somewhat frustrated, and very likely insolent and frank comment (on my part), with a no doubt bewildered Math teacher. Which I suspect would have included a comment along the lines of *"Why do we need to learn this stuff anyway? We're never going to need to use it in Real Life!"*

So here am I, eating humble pie, with an apology to all Mathematics 1 & 2 teachers out there. I was wrong!! And now I've got to work out how to solve my Maths problem on my own!

However, before I tackle that, I'm going to reproduce a table in a desperate attempt to delay the inevitable.

Table of Prior probability

Case	Distribution West , East	Prior Probability
A	K76 , -	11%
B	K7 , 6	13%
C	K6 , 7	13%
D	76 , K	13%
E	K , 76	13%
F	7 , K6	13%
G	6 , K7	13%
H	- , K76	11%

Table 1

For the purists among us, this second table (below) takes into account the fact that the first (Heart) trick has been taken and both Defenders' hands have been reduced to twelve cards (courtesy of SuitPlay).

Case	Distribution West , East	Prior Probability
A	K76 , -	10.87%
B	K7 , 6	13.04%
C	K6 , 7	13.04%
D	76 , K	13.04%
E	K , 76	13.04%
F	7 , K6	13.04%
G	6 , K7	13.04%
H	- , K76	10.87%

Table 2

Onward I go, to the inevitable.

Posterior Probability

Calculating posterior probability is basically an application of Bayes' Theorem and relates directly to Bayes' Rule, which is; "**posterior odds equals prior odds times the likelihood ratio"**

To be more specific; for an arbitrary event A (like dropping the singleton King offside) given a known event B (like East following with a low club after the Queen is led from Dummy), provided that $P(B) \neq 0$,

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Or, $P(A|B) \propto P(A)P(B|A)$

Please note that I intend using the prior probabilities from Table 1 simply because they start out as rational percentages, in an attempt to keep things as clean as possible. Should you wish to, readers can substitute the values from Table 2 into the formulae in order to discern the absolute true percentages.

Righto, for my burning question I'm going to use the expanded equation in order to explain as clearly as I can how the conclusion is reached. Additionally we have the lucky opportunity of being able to use the binary form of the equation because we're playing bridge, which means there should only be one King of Clubs (it can't be in any more than one defender's hand on any occasion).

We can see the prior probability of West holding the King singleton, which we'll call $P(W_{SK})$

$P(W_{SK}) = 0.13$

Ergo, the prior probability of West not holding the King singleton is 1 – 0.13

$P(\neg W_{SK}) = 0.87$

We also know that, whenever West has the singleton King, East must have both of the outstanding low Clubs and hence must play Low;

$P(E_L | W_{SK}) = 1$

What about when West doesn't have the singleton King? We can see from Table 1 there are only 63 out of the 87 in 100 cases, where West doesn't have the singleton King that East can actually play a low Club. The remaining 24 cases in 87 where East can't play low are; when East is void (Case A, 11%) and when East holds the singleton King (Case D, 13%)

$P(E_L | \neg W_{SK}) = \frac{0.63}{0.87}$

The probability that West holds the singleton King after East follows low;

$$P(W_{SK}|E_L) = \frac{P(E_L|W_{SK}) \cdot P(W_{SK})}{P(E_L)}$$

Or;

$$P(W_{SK}|E_L) = \frac{P(E_L|W_{SK}) \cdot P(W_{SK})}{P(E_L|W_{SK}) \cdot P(W_{SK}) + P(E_L\,|\neg W_{SK}) \cdot P(\neg W_{SK})}$$

$$= \frac{1\times 0.13}{1\times 0.13 + \frac{0.63}{0.87}\times 0.87}$$

$$= \frac{0.13}{0.76}$$

$$P(W_{SK}|E_L) \approx 17.1\%$$

Finally an answer! Declarer has a 17.1% chance of dropping the King singleton offside after East follows low. The odds went up! Only a little, but they went up all the same.

Okay, so what about the original line, running the Queen? What happens there?

Well, we know (from our table, if not automatically) that the prior probability of the King onside is a fifty-fifty. Let's call it the King with East, $P(K_E)$ and while we're at it we'll call the King with West $P(K_W)$

$P(K_E) = 0.5$

$P(K_W) = 0.5$

We now want to know what the likelihood is of East holding the King after the fact that East has followed low to the call for the Queen from Dummy. So we need to know the cases where East can play low whenever s/he has the King. We can see from Table 1 that East can only play low on 37 of the fifty cases where s/he holds the King. The other 13 cases are where the King with East is held singleton (Case D).

$P(E_L|K_E) = \frac{0.37}{0.5}$

This time we're going to use a shortcut to determine the prior probability of East playing low, because we can actually get it fairly easily by adding up the cases where East can play low from our table. By adding cases (B+C+E+F+G+H) from Table 1 we can see that;

$P(E_L) = 0.76$

And this probability will remain the same for the same calculation for $P(K_W)$ later because $P(E_L)$ is actually independent of where the King is. It's simply a reflection of East always playing a choice of low Club whenever s/he can, which incidentally can be checked by artificially reducing $P(E_L)$ simulating random voluntary hops with the King onside which, due to its inverse proportionality increases Declarer's likelihood of dropping West's King Singleton in our first calculation. So it's probably not such a clever choice of application of the 'Principle' (for anyone who was wondering).

The probability that East has the King, when East plays low to the Queen called from Dummy;

$$P(K_E|E_L) = \frac{P(E_L|K_E) \cdot P(K_E)}{P(E_L)}$$

$$= \frac{\frac{0.37}{0.5}\times 0.5}{0.76}$$

$P(K_E|E_L) \approx 48.7\%$

Surprised? The likelihood of *successfully* running the Queen has reduced, albeit slightly, now that East has played low on the lead from Dummy. Maybe some of our more experienced readers might not have been surprised at all, but let's now see what the likelihood is of West having the King either doubleton or singleton. It should be 51.3% since that's the only other possible outcome?

Now the likelihood of East playing low whenever West has the King is slightly higher because the three cases where East can play low are made up of 2:1 breaks (Cases B,C, & E) with the one case remaining where East cannot play a Club at all (Case A); so

$$P(E_L|K_W) = \frac{0.39}{0.5}$$

And;

$$P(K_W|E_L) = \frac{P(E_L|K_W) \cdot P(K_W)}{P(E_L)}$$

$$= \frac{\frac{0.39}{0.5} \times 0.5}{0.76}$$

$P(K_W|E_L) \approx 51.3\%$

Okay, I think? Have we accidentally established the fact that once East follows low, there's now a slightly less than even chance that the finesse is on? Yes we have, we've just proven that the King is now more likely to be with West whenever East plays low, but of course, that isn't the end of it just yet. However, at this point I'm going to get a little side-tracked again.

Some of you might be wondering how a more astute player may have already been able to guess that the King offside was going to be 51.3% before we'd even done the second and third calculations. Well let's take another look at table 1, but this time with the newly calculated probabilities that we now know.

Table of Posterior probability

Case	Distribution West , East	Probability
A	K76 , -	0%
B	K7 , 6	17.1%
C	K6 , 7	17.1%
D	76 , K	0%
E	K , 76	17.1%
F	7 , K6	17.1%
G	6 , K7	17.1%
H	- , K76	14.5%

Table 3

It should be noted that even though we are referring to this table as posterior, since it is calculated from the knowledge that East has played a low card, it is now also a table of prior probabilities, and it only remains accurate prior to West's play to the second trick! Hence the reason we tend not to do these calculations on a continual basis. Imagine how long it might take to play a single hand if we all started trying to calculate this stuff in our head *every time* an opponent played a card?

We can see from our new table that Cases A and D can no longer happen and that there are three equal cases remaining where West could hold the King (B,C, & E). We know from Bayes' Rule that all of these conditions increase proportionally after the occurrence of East's following low, so the total likelihood of West holding the King had to be three times the likelihood of West holding the King singleton. We can also see that by deduction, West is twice as likely to hold the doubleton King than hold the singleton King, purely due to the ratio of available Cases (B & C vs. E), and the probability of King Doubleton with West P($W_{DK}|E_L$) is very easy to extrapolate from our previous calculations.

$$\begin{aligned} P(W_{DK}|E_L) &= P(K_W|E_L) - P(W_{SK}|E_L) \\ &= 0.513 - 0.171 \end{aligned}$$

$$P(W_{DK}|E_L) \approx \ 34.2\%$$

But the Text Book says I should *finesse*?

Now why would a reputable tome on Bridge (the Encyclopaedia), and my favourite Bridge software, both be telling me to run the Queen when the odds are less than a fifty-fifty chance? Good question! Is the advice our Declarer got on going for the drop actually correct? Well it might be, but I'm still pretty sure it isn't right on this particular occasion. Though it may seem like a fair proposition, and I guess in extension a fair argument, that at this point we might be prepared to believe that going for the drop maximises our chances after East has played a low Club? Well let's take a look at the case for the drop.

We might argue that once the Queen has been led and East has followed low, that we might as well have a second bite by hopping the Ace and trying to drop the King?

What does it matter if West has the King doubleton? The finesse isn't on in any of these cases anyway? (Cases B & C)

Well this argument is absolutely right, but the particular issue with it is *that it is also absolutely immaterial!* The problem with this argument is that it holds equally true for the situation where a Declarer *would* choose to finesse. Let's look at it from the other angle.

What does it matter if West has the King doubleton? The King dropping isn't on in any of these cases anyway? (Cases B & C!)

Including the case where West holds the King doubleton in our reason for making one play, or the other, is akin to what statisticians and many expert bridge players refer to as *"The Monty Hall Trap".* If you aren't familiar with this paradox you should look the following three puzzles up online. They are; "Bertrand's Box Paradox", "Three Prisoners Problem", & the "Monty Hall Problem". All of which are documented in Wikipedia. Sorry, I'm not going to go into these here. This thing is painfully long enough as is! The only thing here of note is that our optimal strategy is the exact opposite of the strategy recommended for the Monty Hall Trap, and that is (this time) to stick with our original line, running the Queen. And to prove it we get to return to our tried and tested technique. Bayes' Theorem!

Bayes' Factors

Back at the end of the subsection on Posterior probabilities we ended up proving that after East follows low there is actually a higher likelihood that West holds the King than East, and we might have been left wondering why the optimal line of play is still to run the Club Queen?

We needed to establish and understand along the way, that West holding the Club King doubleton is immaterial to the two choices we have; to run the Queen, or to hop up with the Ace? We can now see that the King doubleton is a red herring in the solving of our problem, because regardless of which choice we make, both strategies will fail whenever the King is second with West. So with that in mind we now need to understand *which is* 'absolutely the best decision', and why? And this is where Bayes' Factors come into play.

So what is a Bayes' Factor exactly? Well for our purposes we can refer to it as a "likelihood-ratio test". We are now going to test the relative likelihood of the finesse succeeding versus the likelihood of playing for the drop succeeding, and fortunately this can be done by simply dividing one of our probabilities by the other; All along I've maintained my opinion that running the queen is the right play, so we are now going to test for that with our "Success of Finesse Ratio", or SFR;

$$SFR = \frac{P(K_E|E_L)}{P(W_{SK}|E_L)}$$

$$= \frac{0.487}{0.171}$$

$$SFR \approx 2.85:1$$

Or;

$$SFR = 74\%:26\%$$

Once we discount the condition we can't control, being the King doubleton with West, we end up with a success ratio of running the Queen of almost 3 to 1 against the drop of the King, *and __that is__ __why__ the text book was right!*

The Painful Bit (Part 1)

What does all of this have to do with Real Card Play?

You've made it all the way to the end of this article and at no point have I talked about much besides probability! How about real life? Let's take another look at the hand.

The Hand(again)

	♠ 8 ♥ A ♦ AJ976 ♣ QT9432	
♠ J653 ♥ Q76542 ♦ 53 ♣ K	N W E S	♠ KQ97 ♥ JT98 ♦ Q84 ♣ 76
	♠ AT42 ♥ K3 ♦ KT2 ♣ AJ85	

Bd. 13, North, All

Again, for the sake of the exercise, I'm assuming a Heart lead from West and Declarer immediately running the Club Queen. At this point what would, or should an *Expert East* do if s/he has no reason to believe N-S hold a ten card suit and *if s/he were holding the Club King doubleton?* Dummy's missing the Jack, so should East rise with the King just in case there's a need to promote partner's doubleton Knave?

Maybe there's slightly more of a need for a good Declarer to rise with the Ace, for exactly that same reason? I don't think Bayes' Theorem will ever be able to calculate for all of these scenarios. No, for those types of problems we need to depend on the "Nash equilibrium", but methinks that one should be saved for another day!

The Painful Bit (Part 2)

Are you a true "Propeller-head" of Bridge??

It pains me to say it, but it's very likely that a dedicated expert of the game would have politely read the article up to the display of the hand, taken one glance, answered the problem, and then moved on. Maybe if you do fall into this category you may have read on either; a) purely out of curiosity; b) you really love statistics; or c) you needed to see that I was going to ultimately produce the correct answer, as you feel it is your sole responsibility to maintain the order of factual accuracy in bridge!

So, what is it about this hand that makes the right answer to our problem so easy for an expert? Well, it all boils down to one single card. And if you haven't figured it out just yet, there was a glaring clue in part one above! *Still not sure?*

It's the reason I didn't want to discount out of hand our Declarer's online advice!

The sole key to the question of whether or not to rise with the Ace is down to *'the Jack of Clubs'.*

If we give East South's Club Jack, and South one of East's low cards, then the correct play for Declarer is to **not run the Queen of Clubs,** but rather;

A. "Play the ace, hoping to drop the singleton king" – 6 tricks (26%), 4 tricks (11%)
B. "Lead small to the queen, *or* (best) lead small to the eight" – 5 tricks (100%)

So, just maybe, the original advice (wherever it came from) was indeed accurate? It may just have suffered from the ill fate of Chinese Whispers somewhere between the originator and our West's lucky Right Hand Opponent.

Of course, we all have the necessary tools to check the validity of this second exercise for ourselves now. As for me, I've got another probability problem I've created.

And it starts with me hazarding a good guess at the likelihood I didn't pay enough attention in High School...

'Til next time,

Jim Coffey