PROBABILITIES & VACANT PLACES By Don Sondergeld 12.07.2013

www.phillipalderbridge.com/columns.html contains Phil Alder's excellent newspaper articles on bridge. However, one of my favorite bridge bloggers is Bob MacKinnon, a Canadian from Victoria B C. You can read his articles in **http://bobmackinnon.bridgeblogging.com/** Bob has a great sense of humor. Bob's Partner: "We had a good result today because I did not bid on hand three." Bob: "You should do that more often!"

Bob emphasizes the difference between **"a priori"** probabilities and **"a posteriori"** probabilities. The terms "a priori" (from the earlier) and "a posteriori " (from the later) are used to distinguish between two types of knowledge: "a priori" knowledge is independent of experience, but "a posteriori " knowledge is dependent on extra information you learn.

A bureau contains 4 drawers. One drawer has a spade, one a heart, one a diamond, and one a club. The "apriori" probability of removing a drawer that contains a diamond is 1/4. Assume you remove a drawer containing a club, and note that it contains a club. If you do not return the drawer to the bureau, the "aposteriori" probability of next removing a drawer containing a diamond is now 1/3. It is not important as to whether the diamond is in the top, middle or bottom drawer. The only thing that is important is that there are now only 3 drawers in the bureau and only one contains a diamond. Similarly, probabilities are not affected by how cards are arranged in each hand. However, probabilities do change as the cards are removed from each hand.

It may seem simple, but this concept is important in bridge. You, as South, are Declarer and the opponents have not bid. You know with certainty the distribution of the four suits in your hand and can guess at the distribution of the suits in your partner's hand. At the end of the first trick, you know exactly your "26 card shape ". If you were 3523 and your partner was 2344, your "26 Card Shape" is 5867 so the opponents must have been 8576. Hearts are trump, trump was led, and everyone followed to the first trick. The remaining places in the opponents hand are 8376 = 24. That is, you know the opponents have 24 cards remaining and their "24 Card Shape", but you don't know which specific card of the 24 missing cards is held by the right or left hand opponent .

If West had overcalled a spade and was supported by East, it is almost certain, the 16 non spades must be distributed 12-5 = 7 in West's hand and 12-3 = 9 in East's hand. Whenever the unknown places are reduced, the odds change. Certainty occurs when there are no unknown places.

Bob points out that many bridge players use "a priori" odds and probabilities all through the play of the hand instead of recalculating the "a posteriori" numbers as new information becomes available.

Vacant Places (also called Vacant Spaces)

You are South and Declarer. West had opened 2S and led a spade. After the first trick you believe East started with 2 spades. You now know that of the remaing cards, West has 7 spaces that hold non spades and East has 11 spaces that hold non spades. The theory of vacant places in bridge states that when the distribution of one or more suits is completely known, the probability that an opponent holds a particular card in any other suit is directly proportional to the number of vacant places remaining in their respective hands .

Example 1. Vacant places can be used in calculating the probability of any split, such as a 2-0 split. In the case where Declarer is missing the K and 3 of a suit, the "a priori" odds equal 52% of dropping the K. Why is it not 50:50? The K may be held by West with 1/2 probability: 13 ways, out of the 26 ways that this card can be dealt to West. Now there are 25 vacant places remaining, and the 3 may go to West or East. But there are 12 ways that it can go to West, and 13 ways that it can go to East. The probability that the 3 goes to West is 12/25. So the probability that West gets both is 1/2 x 12/25 = 6/25 or 24%. The probability that East gets both is also 24%, and the probability of a 1-1 split is 52%.

(The same probability can be determined by using mathematical combinations. The probability that West has both cards equals the combination of 24 cards taken 13 at a time divided by the combination of 26 cards taken 13 at a time.)

Example 2. South is the Declarer in 4 Spades missing the King and the 3. If South won the opening lead of the King of hearts, the opponents now have 24 cards remaining. The odds have now improved slightly of a 1-1 split. The probability that West has both the K and the 2 of spades has dropped from 24% to 1/2 x 11/23 to about 23.9%. So the 1-1 split is now 52.2%. The K can be in the 11 remaining vacant places in the West or in the 12 vacant places in the East. In the absence of any helpful information, South should still play for the split.

(The same probability can be determined by using mathematical combinations. The probability that West has both equals the combination of 22 cards taken 12 at a time divided by the combination of 24 cards taken 12 at a time.)

Example 3. South is the Declarer in 4 Spades missing the King and the 3. East had preempted 3 Hearts and South had won the opening lead of the King of hearts in his hand with his singleton Ace. North also had a singleton heart. Think of East and West as two parking garages. 3 of the 12 parking places in West's hand and 6 of the 12 parking places in East's hand contain hearts. There are 9 parking places vacant in West's garage to park the two missing spades. There are only 6 parking places vacant in East's garage to park the two missing spades . The odds are 9-6 the King is in the West. It would appear the 1-1 split is about 52%. But the 9-6 vacant place relationship suggests a finesse.

Example 4. We have all heard about "8 ever, 9 never". Never say never. (One bridge player was asked why he did not lead his singleton 9 in his partner's bid suit against a slam, replied: "8 ever, 9 never!") But, seriously, consider the case where Declarer and Dummy have 9 spades, missing the 2 3 4 Q. There are times when Declarer should finesse for the Queen through the hand with the greater number of vacant places rather than playing for the drop.

If you want to learn more, read Robert F MacKinnon's 2010 book titled Bridge, Probability, & Information. You don't need to understand Bayes Theorem to be a good bridge player. In watching TV you only need to know which buttons to push and in what sequence. So when you play bridge keep an open mind and think about vacant places.

Shape People know what 38 24 34 means in describing a woman's shape. Bridge players know what x is in the following sequences: 334x 820x 640x 887x 599x 986x. x =3. The first three sets equal a "13 Card Shape" in one hand and the next three equal the known "26 Card Shape" in two hands (Declarer +Board or either Defender + Board).

A beginner often pull trump. As Declarer, he or she looks at the combined number of trump contained in his or her hand and in the Dummy and subtracts that from 13 to learn how many trump cards are outstanding. But it is also important to determine how many cards are outstanding in each of the other three suits.

One Hand Patterns There are 635,013,559,600 hands a person can be dealt in bridge. The 39 hand patterns can by classified into four hand types: balanced hands, three suiters, two suiters and single suiters. The table below gives the "a priori" likelihoods of being dealt a certain hand-type. 4432 and 4423 have similar "13 Card Shape".

Similar Shape	%		Similar Shape	%
4432	**21.6**		5521	3.2
5332	15.5		4441	3.0
5431	12.9		7321	1.9
5422	10.6		6430	1.3
4333	10.5		5440	1.2
6322	5.6		5530	0.9
6421	4.7		Other 25	3.7
6331	3.4		Total	100.0

<u>Two Hand Patterns</u> There are 495,918,532,948,104 hands that can be dealt to one person and his partner. There are 104 hand patterns for these two hands combined. If you are Declarer and your shape is 5422 and Dummy is 3442, you are holding an 8864 hand pattern. Defenders therefore have a 5579 distribution. The thrust of this article is to point out that Declarer can recalculate certain odds as play continues. Declarer knows his side's "26 Card Shape", so he also knows the "26 Card Shape" of the opponents.

Our Side	Opponents	%		Our Side	Opponents	%
8765	5678	**23.6**		7775	6668	5.2
7766	6677	10.5		9755	4688	4.9
9764	4679	7.3		8864	5579	4.9
9665	4778	6.6		9854	4589	4.1
8774	5669	6.6		8855	5588	3.3
8666	5777	5.2		Other 93		17.8

The one hand patterns contain 13 cards: i) three suits each with an odd number of cards and one suit with an even number of cards or ii) three suits each with an even number of cards and one suit with an odd number of cards

The two hand patterns total 26 cards. The opponents combined holdings contain: i) all even numbers of cards in each suit, ii) all odd number of cards in each suit, or iii) an even number of cards in two suits and an odd number of cards in two suits. Declarer can determine exactly which pattern is held by the opponents. That knowledge coupled with any bidding by the opponents and subsequent play can then be used to help determine how the individual suits are split.

<u>Bill Butler</u> The probabilities of getting the 39 hand patterns for one hand and the 104 hand patterns for two hands are shown in Bill Butler's web page: **<u>http://www.durangobill.com/BrSuitStats.html</u>** Bill is from Durango, Colorado. Also, details on the calculations involving "Combinatorics" are contained in his web pages.